

Scattering of Gauge, Matter, and Moduli Fields from Intersecting Branes

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Abstract

We calculate various tree-level (disk) scattering amplitudes involving gauge, matter and moduli fields in type *IIB* toroidal orbifold/orientifold backgrounds with *D9*, *D5* respectively *D7*, *D3*-branes or via *T*-duality *D6*-branes in type *IIA* compactifications. In type *IIB* the *D*-branes may have non-vanishing fluxes on their world-volume. From these results we extract the moduli and flux dependence of the tree-level gauge couplings, the metrics for the moduli and matter fields. The non-vanishing fluxes correspond in the *T*-dual type *IIA* description to intersecting *D6*-branes. This allows us to determine the moduli dependence of the tree-level matter field metrics in the effective action of intersecting *D6*-brane models. In addition we derive the physical Yukawa couplings with their correct normalization.

1. Introduction

Four-dimensional superstrings constitute at the moment the best candidates for unification of all interactions. In trying to use these theories to describe the observed physics or the measurements in future accelerator experiments like the LHC, it is of fundamental importance to obtain the low-energy field theory of each given class of 4-D string. In this context the deviation of the minimal supersymmetric standard model (MSSM), the issue of supersymmetry breaking, mass generation, the search for supersymmetric particles, rare decays, flavor changing neutral currents, the search for extra dimensions and gauge coupling unifications will be central topics.

About ten years ago this program was pursued with great detail and also some success (see e.g [1–4]) for 4-D heterotic string vacua [5]. More recently it became evident that open string constructions and most notably intersecting brane worlds [6,7,8] offer excellent opportunities for realistic string phenomenology (see also [9] for a review). Within these string compactifications, in general a hierarchy of dimensions supporting different kinds of massless fields is opening up: first, there is the closed string sector in the entire ten-dimensional space, which contains besides the gravitational fields also the geometric scalar moduli fields of the internal compact space. Second, open strings provide the standard model gauge fields, living on the $p+1$ -dimensional world volumes of Dp -branes, and third, the standard model matter fields correspond to open string excitations which are located at the even lower-dimensional various intersections of the D -branes in the internal six-dimensional space. Then the fermion spectrum is determined by the intersection numbers of certain homology-cycles in the internal space, as opposed to the heterotic strings, where the number of generations was given by the Euler characteristic in the simplest case.

Some results on the low-energy effective action of intersecting brane world models were already obtained in the past: the effective scalar potential is needed to discuss the question of stability of intersecting brane world models [10]; tree-level gauge couplings [11] and one-loop gauge threshold corrections [12] in supersymmetric intersecting brane world models were calculated, and the question of gauge coupling unification was addressed in [13,14]; using the $N=1$ type I – heterotic string-string duality, some results for the matter field Kähler metric and for soft-SUSY breaking terms in the effective action were obtained in [15]; finally effective Yukawa couplings [16,17,18] and higher point scattering [19], relevant for flavor changing neutral currents [20] and proton decay [21], were also investigated. However, the amplitudes discussed in all those references involve only open strings inserted at the boundary. In certain cases the amplitude may be easily deduced from known closed string results. More concretely, the open string vertex operators inserted at the boundary depend only holomorphically on the world-sheet coordinates. If the full open string vertex operator or only its internal part represents just half of an analogous closed string vertex operator, for that particular piece of the full correlator a similar computation on the

sphere may be borrowed. The latter involves both the holomorphic and anti-holomorphic side and the open string result may be obtained by essentially “taking the square root”. However already in the case of scalar matter field vertex operators, this simple procedure fails. E.g. a scattering of four matter fields on the sphere like in heterotic string theory assumes a completely different momentum expansion than the same amplitude with matter fields represented by open strings. To conclude, generically one should not expect a closed string computation in any simple way to be related to a scattering amplitude involving open string states from the boundary. As we shall see, this statement will be strengthened when considering disk amplitudes involving in addition also closed string states from the bulk. These bulk states in particular include all closed string moduli of the underlying compact fields, namely the Kähler moduli T^j as well as the complex structure moduli U^j . Therefore, deriving the moduli dependent gauge coupling constants for the open string gauge fields or the moduli dependent Kähler metrics for the open string matter fields, mixed open/closed string scattering amplitudes have to be computed. As we will see, here subtle issues arise, which are quite different from heterotic string amplitudes.

The aim of this paper is to explicitly compute several disk scattering amplitudes involving open string gauge and matter fields as well as closed string moduli fields from intersecting branes and deduce from them directly relevant parts of the open/closed string 4-D effective, low-energy field theory. We will perform the computation of the open/closed string scattering amplitudes in the type *IIB* (\mathcal{F} -flux picture) as well as in the T -dual type *IIA* picture (angle picture). In type *IIB* we are dealing with $D9$ -branes wrapped on a six-dimensional torus with open string magnetic \mathcal{F} -flux turned on. Therefore the $D9$ -branes have in general mixed Dirichlet and Neumann boundary conditions on the torus, which becomes non-commutative in this way. Via T -duality on three directions of the torus the $D9$ -branes become $D6$ -branes, being wrapped around 3-cycles of the dual torus and intersecting each other on various points of the dual torus, with intersection angles that are in one-to-one correspondences with the *IIB* \mathcal{F} -fluxes. All our disk amplitudes are valid for arbitrary flux values, i.e. arbitrary intersection angles. However when extracting from the amplitudes the low-energy effective action we mainly concentrate on D -brane configurations which preserve 4-D $N=1$ space-time supersymmetry. In type *IIB* this means that the fluxes have to be chosen in such a way that essentially the D -branes have either $D9$ - or $D5$ -brane (or $D7/D3$ -brane) boundary conditions. On the T -dual other side, the $D6$ -branes have to satisfy certain angle conditions in order to be $N=1$ space-time supersymmetric.

It is well known [22] that any $N=1$ supergravity action in four space-time dimensions is encoded by three functions, namely the Kähler potential K , the superpotential W , and the gauge kinetic function f . When such an effective action arises from a higher dimensional string theory these three function usually depend (non-trivially) on moduli fields describing the background of the present string model. It is the aim of this article

to determine the moduli dependence of the gauge kinetic function f and of the Kähler potential K from string tree-level scattering amplitudes. One-loop corrections will be discussed elsewhere.

In the first part of the paper we will compute those scattering amplitudes between open string gauge bosons and closed string moduli fields that are relevant for the moduli-dependent, effective 4-D gauge couplings. This is important for the issue of gauge coupling unification and, in supersymmetric compactifications, for the computation of the soft gaugino masses. The holomorphic gauge kinetic function has been determined for supersymmetric intersecting brane world models, in [11] at string tree-level via dimensional reduction of the Born-Infeld action and in [12] at one-loop by explicit string calculations. Our results indeed here confirm the findings of [11] for supersymmetric D -brane configurations.

Secondly, we compute the kinetic energy terms in the effective action for open string matter fields. Being moduli-dependent quantities, these are needed for the correct normalization of the (physical) Yukawa couplings, for the matter field scalar potential and for the soft scalar masses (squarks and sleptons). In intersecting brane world models chiral fermions stem from open strings stretched between two intersecting branes of angles $\theta = (\theta^1, \theta^2, \theta^3)$. For supersymmetric angles, these fermions come in $N=1$ chiral multiplets, whose lowest component is the scalar matter field C_θ . From the point of view of the open string conformal field theory, the open fields C_θ obey mixed boundary conditions and are associated to twist fields. In addition there may be untwisted matter fields C_i , which come in $N=4$ multiplets for the case of toroidal models.

The Kähler potential for the moduli M and matter fields C_i, C_θ reads up to second order in the matter fields:

$$K = \hat{K}(M, \bar{M}) + \sum_{\substack{\text{twisted} \\ \text{matter } \theta}} G_{C_\theta \bar{C}_\theta}(M, \bar{M}) C_\theta \bar{C}_\theta + \sum_{\substack{\text{untwisted} \\ \text{matter } i, j}} G_{C_i \bar{C}_j}(M, \bar{M}) C_i \bar{C}_j + \mathcal{O}(C^4) . \quad (1.1)$$

The index θ collectively denotes a given matter field coming from one pair of intersecting branes of angles θ . It is the purpose of this article to calculate the metrics $G_{C_\theta \bar{C}_\theta}$ and $G_{C_i \bar{C}_j}$ for intersecting brane worlds from various disk amplitudes involving matter and moduli fields. Untwisted matter field metrics for $N=1$ type I compactifications on toroidal orbifolds with parallel $D9$ -branes without fluxes have been determined in [23]. In addition we will also calculate open string disk amplitudes that describe the scattering of four matter fields. Performing suitable factorization limits these amplitudes contain useful informations on the physical Yukawa couplings and in this way on the (holomorphic) trilinear superpotential which reads:

$$W = \sum_{\alpha, \beta, \gamma} W_{\alpha\beta\gamma}(t^l) C_\alpha C_\beta C_\gamma . \quad (1.2)$$

Our string scattering computations for the matter field metrics substantially extend and improve the claims made in [15]. The latter relies on arguments based on $N=1$ type I – heterotic duality in $D = 4$. This is a strong–weak coupling duality involving a mixing of the dilaton with the other moduli fields. Therefore a comparison of heterotic results on the sphere with a similar type I result from the disk is to be questioned. Moreover, we shall see that the moduli dependence is derived from a disk amplitude involving both open strings from the brane and closed strings from the bulk. This coupling has no obvious relation to a pure closed string coupling on the sphere. In particular, this disk coupling is not just a square root of a similar coupling on the sphere. Furthermore, taking into account the problems discussed in [24], it is a challenging task to find the correct duality map of fields in the effective actions of $N=1$ heterotic and type I theories.

The paper is organized as follows. In the next section we review the aspects of open string conformal field theory which will become relevant for the computation of the various disk amplitudes. Special emphasis is given to D –branes with mixed boundary conditions (type IIB picture), respectively intersecting D –branes (type IIA). Section 3 is devoted to the derivation of the tree level effective gauge couplings. First, we compute the gauge couplings from the Born-Infeld action with fluxes in type IIB , including also the known expressions [11] of the holomorphic f -function for supersymmetric $D9$ – and $D5$ –branes without fluxes. These results are nicely reproduced by the scattering of two open string gauge fields with one closed string modulus, where one has to make a careful transformation between the string moduli, referring to string vertex operators, and the (supergravity) field theory moduli fields. Analogous results are obtained for rotated $D6$ –branes in the T –dual type IIA picture. In sections 4 and 5 we present our results on the scattering of two open string matter fields and one or two closed string moduli. First, in section 4, we derive the amplitudes and the metric for the untwisted matter fields in the presence of fluxes on the branes. After that in section 5 we discuss the twist field open string vertex operators and the associated disk amplitudes and compute the twist field matter metrics. Finally in section 6 we compute the scattering amplitudes of four open string matter fields which are needed for the derivation of the physical Yukawa couplings. Some conclusions are presented in section 7. In appendix A we consider scattering of two closed string moduli fields off Dp –branes. These results allow to determine the Kähler potential for the closed string moduli fields. In appendix B we calculate disk amplitudes involving two matter fields from the boundary and one massless bosonic closed NS –string state from the supergravity multiplet.

2. Disk amplitudes involving open and closed string states

In this subsection we review some technical details occurring in open string disk calculations. The world-sheet diagram of a string S -matrix describing the interaction of open and closed strings at (open string) tree-level can be conformally mapped to a surface with one boundary. The latter may be described by a disk, which is conformally equivalent to the upper (complex) half-plane $\mathcal{H}_+ = \{z \in \mathbf{C} \mid \text{Im}(z) > 0\}$. The open string states are inserted by vertex operators at the boundary of the disk. On the other hand, the closed string vertex operators are inserted inside the disk. In theories with D -branes massless fields like gauge fields, Wilson line moduli or matter fields originate from open string excitations living on the D -brane world-volume. Hence the boundary of the disk diagram is attached to the D -brane world-volume. On the other hand, the closed strings, representing *e.g.* the graviton, dilaton and metric moduli live in the bulk and are inserted in the bulk of the disk. Disk scattering of closed massless strings from the supergravity multiplet in the presence of D -branes has been addressed in the past in Refs. [25–31]. It is one of the purposes of this article to extend these works to moduli fields.

The disk, represented as upper half plane \mathcal{H}_+ , may be obtained from the full complex plane representing the sphere, through a \mathbf{Z}_2 projection $z \mapsto \bar{z}$. Apart from the usual correlators on the sphere

$$\begin{aligned}\langle X^a(z_1) X^b(z_2) \rangle &= -g^{ab} \ln(z_1 - z_2) , \\ \langle \psi^a(z_1) \psi^b(z_2) \rangle &= \frac{g^{ab}}{z_1 - z_2} , \\ \langle \phi(z_1) \phi(z_2) \rangle &= -\ln(z_1 - z_2)\end{aligned}\tag{2.1}$$

for the bosonic fields X^a , fermionic fields ψ^a and ghost field ϕ , this projection implies an interaction between the left-moving and right moving closed string fields (*cf. e.g.* [28]):

$$\begin{aligned}\langle X^a(z_1) \tilde{X}^b(\bar{z}_2) \rangle &= -D^{ab} \ln(z_1 - \bar{z}_2) , \\ \langle \psi^a(z_1) \tilde{\psi}^b(\bar{z}_2) \rangle &= \frac{D^{ab}}{z_1 - \bar{z}_2} , \\ \langle \phi(z_1) \tilde{\phi}(\bar{z}_2) \rangle &= -\ln(z_1 - \bar{z}_2) .\end{aligned}\tag{2.2}$$

The matrix D^{ab} depends on whether Dirichlet or Neumann boundary conditions are imposed on the open string fields attached to the Dp -brane:

$$D^{ab} = \begin{cases} g^{ab} , & \text{tangent indices} , \\ -g^{ab} , & \text{normal indices} . \end{cases}\tag{2.3}$$

We shall calculate disk correlators involving N_o open strings inserted at the boundary $z = \bar{z}$ of the disk and N_c closed strings from the bulk. More precisely, we concentrate

on the following kinds of disk amplitudes (i) $N_o = 4$ $N_c = 0$, (ii) $N_o = 2$, $N_c = 1$, (iii) $N_o = 0$, $N_c = 2$ and (iv) $N_o = 2$, $N_c = 2$. The case (i) is much easier to handle than the other cases. In the case (i) only massless open string states are inserted at the boundary of \mathcal{H}_+ . The latter describe massless fields like gauge fields or Wilson lines living on the D -brane world volume. Their vertex operators involve only holomorphic fields and *Eq.* (2.1) describe their interaction. The same holds for charged matter fields living on the D -brane world volume (or intersections thereof). On the other hand, if $N_c \neq 0$ also anti-holomorphic fields are involved and produce non-trivial interactions (2.2) between holomorphic and anti-holomorphic fields. This additional mixing may simply be taken into account with the so-called "doubling trick". All left-moving fields of the closed string vertex operator are multiplied by the matrix D :

$$\tilde{X}^i(\bar{z}) \rightarrow D^i_j X^j(\bar{z}) \quad , \quad \tilde{\psi}^i(\bar{z}) \rightarrow D^i_j \psi^j(\bar{z}) \quad , \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}) . \quad (2.4)$$

Their interactions (2.2) then follow from the sphere correlators (2.1) after taking into account the matrix D^{ij} in (2.4). Essentially this means that the world-sheet is doubled to a sphere with left- and right-moving parts of one closed string vertex (inserted at the point $z \in \mathcal{H}_+$) with momentum

$$q^{\parallel} := \frac{1}{2}(q + Dq) \quad (2.5)$$

assumed to constitute two open string vertex operators. One of the open strings is inserted with momentum $q/2$ on the sphere at z and the other with momentum $Dq/2$ at the location \bar{z} . Hence a disk scattering of N_o open strings inserted at the boundary $z = \bar{z}$ and N_c closed strings from the bulk is similar to a string scattering of $N_o + 2N_c$ open strings on the double cover. There are also differences, namely the Chan-Paton factors, the position on the world-sheet and extra constraints on the momenta and polarizations.

Let us stick to our case (ii) involving two open strings of momenta p_1 and p_2 , respectively and one closed string of momentum q . Only the momentum along the direction of the D -brane is conserved:

$$p_1 + p_2 + q^{\parallel} = 0 . \quad (2.6)$$

We may write the relevant closed string momentum $q^{\parallel} = \frac{1}{2}(q + Dq)$. According to the discussion above, we may split the closed string into two open strings. Therefore, we have four open strings with momenta $p_1, p_2, \frac{1}{2}p$ and $\frac{1}{2}Dp$, respectively. Note, that in the four open string vertex operators the doubled momenta k_i enter:

$$k_1 = 2p_1 \quad , \quad k_2 = 2p_2 \quad , \quad k_3 = q \quad , \quad k_4 = Dq . \quad (2.7)$$

With these momenta k_i the momentum conservation (2.6) may be written:

$$k_1 + k_2 + k_3 + k_4 = 0 \quad (2.8)$$

like in an usual four particle scattering process. Since we consider massless strings only, *i.e.* $p_1^2 = p_2^2 = q^2 = 0$, we have $k_i^2 = 0$. We also introduce the kinematic invariants ($s + t + u = 0$):

$$s = k_1 k_2 \quad , \quad t = k_1 k_3 \quad , \quad u = k_1 k_4 \quad . \quad (2.9)$$

However, as already anticipated above, all four momenta are not independent. This has the consequence [27]:

$$s = -2t \quad , \quad u = t \quad , \quad (2.10)$$

which means, that a scattering of two open strings with one closed string allows only for one independent kinematic variable. We shall always work with the momenta k_i , *i.e.* we use k_i in the open string vertex operator, while the closed string vertex carries the momentum q .

So far we have only discussed the case of pure Neumann or Dirichlet boundary conditions. These two cases are encoded in the matrix D , introduced in (2.3). Quite generally, including the case of mixed boundary conditions, which are governed by a non-trivial background flux \mathcal{F} on the D -brane, the matrix D is given by:

$$D = -g^{-1} + 2 (g + \mathcal{F})^{-1} \quad , \quad (2.11)$$

as follows by comparing the correlators (2.1) and (2.2) with the analogous expressions in [32,33]. The matrix D captures the special cases $\mathcal{F} = 0$ and $\mathcal{F} \rightarrow \infty$ corresponding to pure Neumann or Dirichlet boundary conditions, respectively. For these two cases the matrix D boils down to (2.3). With the open string metric G and the non-commutativity parameter Θ

$$\begin{aligned} \Theta^{-1} &= -(g + \mathcal{F})^{-1} \mathcal{F} (g - \mathcal{F})^{-1} \quad , \\ G^{-1} &= (g + \mathcal{F})^{-1} g (g - \mathcal{F})^{-1} \quad , \end{aligned} \quad (2.12)$$

introduced¹ in [33], we may also write²:

$$D = -g^{-1} + 2 G^{-1} + 2 \Theta^{-1} \quad . \quad (2.13)$$

3. Tree-level scattering of two gauge fields and one modulus

In this section we shall investigate quite generally the tree-level gauge couplings of Dp -branes wrapped on $p - 3$ -cycles ($p \geq 3$). As we shall demonstrate in subsection 3.1 these couplings may be easily obtained by dimensionally reducing the Born-Infeld action of the Dp -brane on the $p - 3$ -cycle \mathcal{C}_{p-3} . But we consider it to be a useful warm up

¹ Compared to [33], we performed the replacement: $2\pi\alpha' B \rightarrow \mathcal{F}$ and $\theta^{-1} \rightarrow 2\pi\alpha' \Theta^{-1}$.

² Note: $D_{\text{symm}} = \frac{1}{2}(D + D^t) = -g^{-1} + 2 G^{-1}$ and $D_{\text{anti}} = \frac{1}{2}(D - D^t) = 2 \Theta^{-1}$.

to derive this result independently by a string scattering amplitude in order to introduce some crucial ingredients necessary for the determination of the matter field metric. We shall determine the tree-level gauge couplings of Dp -branes with fluxes expressed by the complex moduli fields describing the internal compactification manifold. This generalizes the discussions in Refs. [34,11]. In particular our formulae capture also intersecting branes with tilted tori. After performing T -duality in n directions, we generically obtain the gauge couplings on tilted $D(p-n)$ -branes. We set up the discussion in type IIB . Hence we consider $D9, D7, D5$ and $D3$ -branes with a $U(N)$ gauge theory on their world-volume. The $D9, D7, D5$ -branes are wrapped on 6, 4, 2-cycles, respectively. In addition, we assume the cycles to be factorizable into 2-cycles $T^{2,j}$, $j = 1, 2, 3$. The discussion takes over to type I after dropping the $D7$ and $D3$ -brane. The type I theory may be obtained from type IIB by an orientifold projection. The world-volume gauge theory on the D -brane sitting on the orientifold plane becomes then $SO(2N)$ or $USp(N)$. In that case, all gauge couplings, derived in the following for $U(N)$ gauge groups, have to be multiplied by a factor of 2, *i.e.* $g_{Dp,SO(2N)}^2 = 2g_{Dp}^2$ [35].

3.1. Gauge couplings of Dp -branes with fluxes (Type IIB picture)

We start with the type IIB superstring in ten space-time dimensions ($D = 10$). Its Einstein term is given by

$$\mathcal{S}_{IIB} = -\frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g_{10}} e^{-2\phi_{10}} R, \quad (3.1)$$

with the dilaton field ϕ_{10} in $D = 10$. On the other hand the dynamics of the massless NS fields on the Dp -brane are described by the Born-Infeld action

$$\mathcal{S}_p = -T_p \int d^{p+1}\xi e^{-\phi_{10}} \text{Tr} \sqrt{-\det(G + B + 2\pi\alpha' F)}, \quad (3.2)$$

with the world volume gauge field strength F . The metric G and the anti-symmetric tensor B are the pull-backs of the bulk tensors to the D -brane world-volume. The Dp -brane tension, describing the coupling of the gauge boson on the brane to the closed string dilaton in the bulk, is given by $T_p = (2\pi)^{-p} \alpha'^{-\frac{1}{2} - \frac{p}{2}}$ (see *e.g.* [35] for details). We compactify type IIB on a six-torus T^6 . The latter is assumed to be a direct product of three two-tori, *i.e.* $T^6 = \otimes_{j=1}^3 T^{2,j}$. Their corresponding metrics g^j

$$g^j = \begin{pmatrix} (R_1^j)^2 & R_1^j R_2^j \cos \alpha^j \\ R_1^j R_2^j \cos \alpha^j & (R_2^j)^2 \end{pmatrix} \quad (3.3)$$

lead to the following complexified Kähler and complex structure moduli:

$$T^j = b^j + i R_1^j R_2^j \sin \alpha^j, \quad U^j = \frac{R_2^j}{R_1^j} e^{i\alpha^j}, \quad j = 1, 2, 3. \quad (3.4)$$

Here b^j is the value internal NS B -field, which i.g. is restricted by the projection and not a dynamical scalar in the theory. As discussed below, in type I the real part of the physical field is given by the axionic scalars a^j that stem from reducing the RR 2-form C_2 on the 2-cycles $T^{2,j}$: $a^j = \int_{T^{2,j}} C_2$. However, some the following formulae take a more suggestive form in terms of the geometric moduli T^j as defined above.

The internal antisymmetric tensors b^j and gauge flux F^j are assumed to be block-diagonal w.r.t. the three tori, too:

$$F^j \equiv \begin{pmatrix} 0 & F^j \\ -F^j & 0 \end{pmatrix} , \quad b^j \equiv \begin{pmatrix} 0 & b^j \\ -b^j & 0 \end{pmatrix} . \quad (3.5)$$

Hence, in the j -th torus $T^{2,j}$ the total internal anti-symmetric background is

$$\mathcal{F}^j = b^j + 2\pi\alpha' F^j = \begin{pmatrix} 0 & f^j \\ -f^j & 0 \end{pmatrix} , \quad (3.6)$$

with the value

$$f^j = b^j + 2\pi\alpha' F^j . \quad (3.7)$$

To obtain a gauge theory in four space-time dimensions, we wrap the Dp -brane ($p = 9, 7, 5, 3$) on a factorizable $p-3$ -cycle \mathcal{C}_{p-3} , which is the product of tori $\mathcal{C}_{p-3} = \prod_{j=1}^{(p-3)/2} T^{2,j}$. Without Wilson lines and additional $U(1)$ gauge fields the internal dimensions decouple from the space-time dimensions. Therefore the $(p+1) \times (p+1)$ matrices G, B and F may be split into the blocks g_6, b_6, F_6 and g, b, F , describing the internal compactification and the $D = 4$ space-time, respectively. The latter shall describe the $D = 4$ space-time, with $b = 0$. The matrix F is the anti-symmetric 4×4 matrix representing the space-time field strength of the gauge field living on the Dp -brane. To this end, after dimensional reduction³ of (3.1) on the six-torus $T^6 = \prod_{j=1}^3 T^{2,j}$ and reducing (3.2) on the \mathcal{C}_{p-3} -cycle specified above we obtain ($p = 3, 5, 7, 9$):

$$\begin{aligned} \mathcal{S}_4 = & - \int d^4x \sqrt{-g} \left\{ \frac{e^{-2\phi_{10}}}{(2\pi) \alpha'^4} R \prod_{j=1}^3 \sqrt{\det(g^j)} \right. \\ & \left. - T_p (2\pi)^{p-3} e^{-\phi_{10}} \sqrt{\det(1 + 2\pi\alpha' F g^{-1})} \prod_{j=1}^{(p-3)/2} |n^j| \sqrt{\det(g^j + \mathcal{F}^j)} \right\} . \end{aligned} \quad (3.8)$$

³ The extra factors of $(2\pi)^6$ and $(2\pi)^{p-3}$ stem from the circumference of each circle.

The integer n^j counts how often the Dp -brane is wrapped around the torus $T^{2,j}$. From this action we may read off the gravitational coupling⁴ $G_N = M_{\text{Planck}}^{-2}$ in four space-time dimensions ($D = 4$)

$$M_{\text{Planck}}^2 = 8 \frac{e^{-2\phi_{10}}}{\alpha'^4} \prod_{j=1}^3 \sqrt{\det(g^j)} = 8 \frac{e^{-2\phi_{10}}}{\alpha'^4} \prod_{j=1}^3 \text{Im}(T^j) = 8 \frac{e^{-2\phi_4}}{\alpha'} , \quad (3.9)$$

i.e. $\kappa_4^{-2} = \frac{e^{-2\phi_4}}{\pi\alpha'}$. Here we have introduced the dilaton field ϕ_4 in $D = 4$ [36]

$$\phi_4 = \phi_{10} - \frac{1}{2} \ln [\text{Im}(T^1)\text{Im}(T^2)\text{Im}(T^3)/\alpha'^3] . \quad (3.10)$$

Furthermore, for the various gauge couplings referring to the Dp -brane wrapped on a \mathcal{C}_{p-3} -cycle we get⁵:

$$\begin{aligned} g_{Dp}^{-2} &= (2\pi)^{-1} \alpha'^{\frac{3-p}{2}} e^{-\phi_{10}} \prod_{j=1}^{(p-3)/2} |n^j| \sqrt{\det(g^j + \mathcal{F}^j)} \\ &= (2\pi)^{-1} \alpha'^{\frac{3-p}{2}} e^{-\phi_{10}} \prod_{j=1}^{(p-3)/2} |n^j| |T_2^j + if^j| \\ &= e^{-\phi_4} (2\pi)^{-1} \alpha'^{3-\frac{p}{2}} \prod_{j=1}^3 \frac{1}{\sqrt{\text{Im}(T^j)}} \prod_{j=1}^{(p-3)/2} |n^j| |T_2^j + if^j| . \end{aligned} \quad (3.11)$$

As we will see in subsection 3.3. it is the last line of (3.11), which is most directly inferred from string amplitudes, *i.e.* disk scattering of two gauge fields and one modulus. Hence we call the moduli fields T^j and U^j appearing above Kähler and complex structure moduli in the *string basis*, respectively. For any N=1 orientifold/orbifold compactification of type I with $D9$ - and $D5$ -branes the gauge couplings take the form (3.11) up to some normalization constant. Without fluxes according to N=1 SUSY in $D = 4$ these couplings have to represent real parts of a holomorphic function, namely the gauge kinetic function. Therefore we introduce new fields s and t^j with their corresponding (real parts) reproducing the $D9$ and $D5$ -brane couplings without fluxes. These fields are called the moduli in the *supergravity field-theory basis* in contrast to T^j and U^j referring to the *string basis*. The real part of the dilaton field s gives the coupling of the gauge fields of the $D9$ brane with wrapping number $n^j = 1$ [37,34]

$$\text{Re } s := g_{D9}^{-2} = (2\pi)^{-1} \frac{e^{-\phi_{10}}}{\alpha'^3} T_2^1 T_2^2 T_2^3 = (2\pi)^{-1} \frac{e^{-\phi_4}}{\alpha'^{3/2}} \sqrt{T_2^1 T_2^2 T_2^3} . \quad (3.12)$$

⁴ The coefficient of R in $D = 4$ is defined to be $\frac{1}{2}\kappa_4^{-2} = (16\pi G_N)^{-1}$, with G_N being the $D = 4$ Newton constant.

⁵ With $T_2^j = \text{Im}(T^j)$, $T_1^j = \text{Re}(T^j)$ and $U_2^j = \text{Im}(U^j)$, $U_1^j = \text{Re}(U^j)$.

The Kähler moduli t^j in the *field-theory basis* follow from wrapping once ($n^j = 1$) the $D5$ on the tori $T^{2,j}$ [37,34]:

$$\text{Re } t^j := g_{D5,j}^{-2} = (2\pi)^{-1} \frac{e^{-\phi_{10}}}{\alpha'} T_2^j = (2\pi)^{-1} \alpha'^{1/2} e^{-\phi_4} \sqrt{\frac{T_2^j}{T_2^k T_2^l}} \quad , \quad (j, k, l) = \overline{(1, 2, 3)} \quad . \quad (3.13)$$

The imaginary parts a, a^j of the fields s and t^j describe the corresponding axions following from the RR form couplings $C_6 \wedge F \wedge F$ on the $D9$ and $C_2 \wedge F \wedge F$ on the $D5$ -brane after integrating the RR forms C_6 and C_2 over the 6-cycle T^6 and 2-cycle $T^{2,j}$, respectively: $a^j = \int_{T^{2,j}} C_2$, $a = \int_{T^6} C_6$ (*cf. Eq. (3.4)*). The Kähler potential for the closed string moduli is [37]

$$\kappa_4^2 \widehat{K}(M, \overline{M}) = -\ln(s + \overline{s}) - \sum_{i=1}^3 \ln(t^i + \overline{t}^i) - \sum_{i=1}^3 \ln(u^i + \overline{u}^i) \quad , \quad (3.14)$$

where $u^i = -iU^i$ are the complex structure moduli.

In the following we shall concentrate on the gauge coupling of $D9$ -branes with non-vanishing fluxes F^j :

$$g_{D9}^{-2} = \frac{e^{-\phi_4}}{(2\pi) \alpha'^{3/2}} \prod_{j=1}^3 \frac{|n^j T^j + m^j|}{\sqrt{\text{Im}(T^j)}} \quad . \quad (3.15)$$

Recall that in this equation the real part of T^j is given in terms of the NS background field b^j , just like for the geometric Kähler modulus in heterotic compactifications. Obviously, the gauge coupling does not depend on the complex structure moduli U^j , in accordance with the general arguments of [38]. The $D = 4$ gauge couplings following for the $D7, D5$ and $D3$ -branes (encoded in (3.11)) may be obtained from this formula by taking the respective limits $f^j = b^j + 2\pi\alpha' F^j \rightarrow \infty$, $n^j \rightarrow 0$, and multiplying the result by factors of $(2\pi)^2$. This limit converts the Neumann boundary conditions of the coordinates of torus $T^{2,j}$ into Dirichlet. In the case of N=1 supersymmetry in $D = 4$ the fluxes f^j , specified in (3.7), have to obey the condition [7,39]

$$\sum_{j=1}^3 \frac{f^j}{\text{Im}(T^j)} = \prod_{j=1}^3 \frac{f^j}{\text{Im}(T^j)} \quad . \quad (3.16)$$

With this constraint the gauge coupling (3.15) becomes the real part of a holomorphic function as dictated by N=1 supersymmetry [40]:

$$g_{D9}^{-2} = |n^1 n^2 n^3| \left| \text{Re} \left(s - \alpha'^{-2} f^1 f^2 t^3 - \alpha'^{-2} f^1 f^3 t^2 - \alpha'^{-2} f^2 f^3 t^1 \right) \right| \quad . \quad (3.17)$$

A similar discussion follows for supersymmetric orientifold/orbifold compactifications of type *IIB* with *D7*– and *D3*–branes⁶.

3.2. Gauge couplings of rotated *D6*–branes (Type *IIA* picture)

Now we perform a *T*–duality in the *y*–directions of each torus $T^{2,j}$. It implies that the type *IIB* moduli T^j, U^j are related to the type *IIA* moduli T'^j, U'^j as follows:

$$T^j = -\frac{\alpha'}{U'^j} \quad , \quad U^j = -\frac{\alpha'}{T'^j} \quad , \quad j = 1, 2, 3 . \quad (3.18)$$

Under this duality the $D = 10$ coupling constant transforms according to:

$$e^{-\phi_{10}} \longrightarrow \frac{e^{-\phi_{10}}}{\alpha'^{3/2}} \prod_{j=1}^3 |U'^j| \sqrt{\frac{T'^j}{U'^j}} . \quad (3.19)$$

Note, that the $D = 4$ dilaton ϕ_4 stays inert under these transformations. The *D9*–brane with the background gauge fluxes \mathcal{F}^j is converted into a *D6*–brane at angles θ^j w.r.t. the *x*–axis. The angle θ^j is given by [39]

$$\tan(\pi\theta^j) = \frac{f^j}{\text{Im}(T^j)} = \frac{1}{\text{Im}(T^j)} (b^j + 2\pi\alpha' F^j) , \quad (3.20)$$

expressed in type *IIB* quantities. After the *T*–duality (3.18) the choice

$$F^j = \frac{1}{2\pi\alpha'} \frac{m^j}{n^j} \quad (3.21)$$

leads to the corresponding type *IIA* relation:

$$\tan(\pi\theta^j) = -\frac{U'^j_1}{U'^j_2} + \frac{m^j}{n^j} \frac{|U'^j|^2}{U'^j_2} = \frac{\tilde{m}^j}{n^j} \frac{|U'^j|^2}{U'^j_2} . \quad (3.22)$$

Here we introduced the shifted wrapping number

$$\tilde{m}^j = m^j - \frac{U'^j_1}{|U'^j|^2} n^j \quad (3.23)$$

⁶ In this setup we have: $\text{Re } s := g_{D3}^{-2} = \frac{e^{-\phi_4}}{2\pi} \frac{\alpha'^{3/2}}{\sqrt{T_2^1 T_2^2 T_2^3}}$, and $\text{Re } t^j := g_{D7,j}^{-2} = \frac{e^{-\phi_4}}{2\pi\alpha'^{1/2}} \sqrt{\frac{T_2^k T_2^l}{T_2^j}}$. The gauge coupling of a *D7*–brane, wrapped on the two tori $T^{2,k}$ and $T^{2,l}$ with the two–form fluxes f^k and f^l becomes: $g_{D7,j}^{-2} = |n^k n^l| |\text{Re}(t^j - \alpha'^{-2} f^k f^l s)|$. The $N=1$ supersymmetry condition demands: $\frac{f^k}{T_2^k} = -\frac{f^l}{T_2^l}$. The imaginary parts a, a^j of the fields s and t^j describe the corresponding axions following from the *RR*–form couplings $C_0 \wedge F \wedge F$ on the *D3* and $C_4 \wedge F \wedge F$ on the *D7*–brane after integrating the *RR*–form C_4 over the 4–cycle $T^{2,k} \times T^{2,l}$: $a^j = \int_{T^{2,k} \times T^{2,l}} C_4$.

describing a tilted torus. The supersymmetry condition (3.16) on the fluxes translates into a relation between the complex structure moduli

$$\sum_{j=1}^3 \frac{\tilde{m}^j}{n^j} \frac{|U'^j|^2}{U_2'^j} = \prod_{j=1}^3 \frac{\tilde{m}^j}{n^j} \frac{|U'^j|^2}{U_2'^j}, \quad (3.24)$$

which after (3.22) becomes a condition on the angles θ^j [7]:

$$\theta^1 + \theta^2 + \theta^3 = 0 \pmod{2}. \quad (3.25)$$

Apparently, the duality action (3.18) converts (3.15) into:

$$g_{D6}^{-2} = \frac{e^{-\phi_4}}{(2\pi)} \prod_{j=1}^3 \frac{|n^j - m^j U'^j|}{\sqrt{\text{Im}(U'^j)}}. \quad (3.26)$$

Evidently, the gauge coupling, being proportional to the volume of the relevant 3-cycle, does not depend on the *IIA* Kähler moduli T'^j . This expression may be directly derived from dimensional reducing the Born–Infeld action (3.2) of a *D6*–brane on a 3-cycle, which is a direct product of three 1-cycles \mathcal{C}_1^j , $j = 1, 2, 3$. Each of this 1-cycle has the wrapping numbers (n^j, m^j) w.r.t. the homology basis of the torus $T^{2,j}$.

The real parts of the type *IIB* field theoretical moduli fields s and t^j become after the *T*–duality transformations (3.18) and (3.19)

$$\text{Re } s' = (2\pi)^{-1} e^{-\phi_4} \frac{\sqrt{U_2'^1 U_2'^2 U_2'^3}}{|U'^1 U'^2 U'^3|}, \quad \text{Re } u'^j = (2\pi)^{-1} e^{-\phi_4} \sqrt{\frac{U_2'^j}{U_2'^k U_2'^l}} \left| \frac{U'^k U'^l}{U'^j} \right|, \quad (3.27)$$

which respectively boil down to $\text{Re } s' = \frac{e^{-\phi_4}}{(2\pi)} \frac{1}{\sqrt{U_2'^1 U_2'^2 U_2'^3}}$ and $\text{Re } u'^j = \frac{e^{-\phi_4}}{(2\pi)} \sqrt{\frac{U_2'^k U_2'^l}{U_2'^j}}$ for rectangular tori $\alpha'^j = \pi/2$. With the additional constraint (3.24) the gauge coupling (3.26) becomes the real part of a holomorphic function:

$$g_{D6}^{-2} = \left| \text{Re} \left(n^1 n^2 n^3 s' - n^1 \tilde{m}^2 \tilde{m}^3 u'^1 - n^2 \tilde{m}^1 \tilde{m}^3 u'^2 - n^3 \tilde{m}^1 \tilde{m}^2 u'^3 \right) \right|. \quad (3.28)$$

3.3. Scattering of two gauge fields and one modulus from *D*–branes with fluxes

In this section we want to check the formulae (3.15) and (3.26) by performing an explicit string calculation. Let us start on the type *IIB* or type *I* side with a stack of *D9*–branes describing the generic gauge group G_a . We allow for non–vanishing fluxes $\mathcal{F}^j = b^j + 2\pi\alpha' F^j$ w.r.t. the internal torus dimensions on which the *D9* is wrapped. As already mentioned before, we may obtain lower dimensional branes by taking the special limit $\mathcal{F}^j \rightarrow \infty$ in some planes j . To gain the full moduli dependence of the tree–level gauge

couplings, we calculate the disk amplitudes $\langle A^a A^a V_{T^j} \rangle$ and $\langle A^a A^a V_{U^j} \rangle$ involving two gauge fields A_μ^a on the boundary and one closed string modulus T^j and U^j , respectively. The latter refer to the j -th subtorus $T^{2,j}$. In the zero ghost picture the gauge field vertex operator is

$$V_{A^a}^{(0)}(z, k) = \lambda^a \xi_\mu [\partial X^\mu + i (k\psi) \psi^\mu] e^{ik_\rho X^\rho(z)} . \quad (3.29)$$

Here λ^a is a Chan–Paton factor in the adjoint of the gauge group G_a , describing the fundamental gauge degrees of freedom at the open string ends. In addition, the polarization ξ_μ has to fulfill $\xi_\mu k^\mu = 0$. Furthermore the T^j and U^j vertex operators in the $(-1, -1)$ ghost picture are given by

$$\begin{aligned} V_{T^j}^{(-1, -1)}(\bar{z}, z, k) &= \frac{1}{2} e^{-\tilde{\phi}(\bar{z})} e^{-\phi(z)} \frac{\partial}{\partial T^j} (g^j + b^j)_{kl} \tilde{\psi}^k(\bar{z}) \psi^l(z) e^{ik_\rho X^\rho(z, \bar{z})} \\ &= \frac{1}{T^j - \bar{T}^j} e^{-\tilde{\phi}(\bar{z})} e^{-\phi(z)} \tilde{\Psi}^j(\bar{z}) \bar{\Psi}^j(z) e^{ik_\rho X^\rho(z, \bar{z})} , \\ V_{U^j}^{(-1, -1)}(\bar{z}, z, k) &= \frac{1}{2} e^{-\tilde{\phi}(\bar{z})} e^{-\phi(z)} \frac{\partial}{\partial U^j} (g^j + b^j)_{kl} \tilde{\psi}^k(\bar{z}) \psi^l(z) e^{ik_\rho X^\rho(z, \bar{z})} , \\ &= \frac{-1}{U^j - \bar{U}^j} e^{-\tilde{\phi}(\bar{z})} e^{-\phi(z)} \tilde{\Psi}^j(\bar{z}) \Psi^j(z) e^{ik_\rho X^\rho(z, \bar{z})} , \end{aligned} \quad (3.30)$$

with the backgrounds⁷ g^j and b^j defined in (3.3). More precisely the vertex operator for the imaginary part T_2^j of T is given by $V_{T_2} = i(V_T - \bar{V}_T)$, which amounts to symmetrizing the vertex operator V_T w.r.t. to the left- and right-movers. The RR vertex operator for the real part of T can be obtained from space-time supersymmetry. In the following we compute the amplitudes for the imaginary part T_2 by using the above operator V_T and its conjugate and summing the two amplitudes at the end of the computation.

Above we have also introduced the complex bosonic and fermionic fields ($j = 1, 2, 3$):

$$\begin{aligned} \bar{Z}^j &= \sqrt{\frac{T_2^j}{2U_2^j}} (X^{2j-1} + U^j X^{2j}) , \quad Z^j = \sqrt{\frac{T_2^j}{2U_2^j}} (X^{2j-1} + \bar{U}^j X^{2j}) , \\ \bar{\Psi}^j &= \sqrt{\frac{T_2^j}{2U_2^j}} (\psi^{2j-1} + U^j \psi^{2j}) , \quad \Psi^j = \sqrt{\frac{T_2^j}{2U_2^j}} (\psi^{2j-1} + \bar{U}^j \psi^{2j}) . \end{aligned} \quad (3.31)$$

In this writing, the Green's functions (2.1) for the internal bosonic fields ∂Z and fermions Ψ take the simple form:

$$\begin{aligned} \langle \partial Z^j(z_1) \partial \bar{Z}^j(z_2) \rangle &= -\frac{1}{(z_1 - z_2)^2} , \quad \langle \partial Z^j(z_1) \partial Z^j(z_2) \rangle = 0 \\ \langle \Psi^j(z_1) \bar{\Psi}^j(z_2) \rangle &= \frac{1}{z_1 - z_2} , \quad \langle \Psi^j(z_1) \Psi^j(z_2) \rangle = 0 . \end{aligned} \quad (3.32)$$

⁷ Note, that for simplicity we assume a compactification on a six-torus T^6 with the latter being a direct product of three single two-tori $T^{2,j}$.

Moreover, the Green's functions (2.2) assume a very compact form, too. First, the D -matrix, introduced in (2.11), takes for the metric g^j and the antisymmetric background $\mathcal{F}^j = b^j + 2\pi\alpha' F^j$ (cf. (3.6)) the form⁸:

$$D^{j,ab} = \frac{2}{(T^j - \bar{T}^j)(U^j - \bar{U}^j)} \frac{1}{(T^j - \bar{T}^j)^2 - 4(f^j)^2} \times \begin{pmatrix} -2|U^j|^2 [4(f^j)^2 + (T^j - \bar{T}^j)^2] & U(2f^j + T^j - \bar{T}^j)^2 + \bar{U}^j(2f^j - T^j + \bar{T}^j)^2 \\ U(2f^j - T^j + \bar{T}^j)^2 + \bar{U}^j(2f^j + T^j - \bar{T}^j)^2 & -2[4(f^j)^2 + (T^j - \bar{T}^j)^2] \end{pmatrix}. \quad (3.33)$$

With this matrix, we obtain⁹ for (2.2):

$$\begin{aligned} \langle \partial Z^j(z_1) \bar{\partial} \bar{Z}^j(\bar{z}_2) \rangle &= -\frac{D^j}{(z_1 - \bar{z}_2)^2} \quad , \quad \langle \partial Z^j(z_1) \bar{\partial} Z^j(\bar{z}_2) \rangle = 0 \quad , \\ \langle \Psi^j(z_1) \widetilde{\Psi}^j(\bar{z}_2) \rangle &= \frac{D^j}{z_1 - \bar{z}_2} \quad , \quad \langle \Psi^j(z_1) \widetilde{\Psi}^j(z_2) \rangle = 0 \quad , \end{aligned} \quad (3.34)$$

with:

$$D^j = \frac{T^j - \bar{T}^j + 2f^j}{T^j - \bar{T}^j - 2f^j} = \frac{T_2^j - i f^j}{T_2^j + i f^j} . \quad (3.35)$$

Obviously, we have $D^j \bar{D}^j = 1$, and

$$D^j = \begin{cases} 1 \quad , & \text{pure Neumann} \quad , \\ -1 \quad , & \text{pure Dirichlet} \quad , \end{cases} \quad (3.36)$$

for boundary conditions of the same sort in both directions of the $T^{2,j}$. These two cases are simply obtained from (3.35) in the limit $f^j \rightarrow 0$ and $f^j \rightarrow \infty$, respectively. Due to internal $U(1)$ charge conservation, correlators involving fields from different planes j vanish for our choice of background, the six-torus T^6 being a direct product of three single two-tori $T^{2,j}$.

First we shall calculate the amplitude:

$$\mathcal{A}_{A^{a_1} A^{a_2} T^j} = \frac{i}{4\pi} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \langle V_{A^{a_1}}^{(0)}(z_1, k_1) V_{A^{a_2}}^{(0)}(z_2, k_2) V_{T^j}^{(-1,-1)}(\bar{z}_3, z_3, q) \rangle . \quad (3.37)$$

We have chosen the modulus vertex operator (3.30) in the $(-1, -1)$ ghost picture in order to guarantee a total ghost charge of -2 on the disk. This requirement is a consequence

⁸ Note, that in two uncompactified directions, the matrix D takes the familiar form: $D^{\mu\nu} = \frac{1}{1+f^2} \begin{pmatrix} 1-f^2 & -2f \\ 2f & 1-f^2 \end{pmatrix}$, which is orthogonal $DD^t = 1$. However, we take $D^{\mu\nu} = \delta^{\mu\nu}$ in the four uncompactified directions.

⁹ Note: $\langle \partial \bar{Z}^j(z_1) \bar{\partial} Z^j(\bar{z}_2) \rangle = -\frac{\bar{D}^j}{(z_1 - \bar{z}_2)^2}$, $\langle \bar{\partial} Z^j(\bar{z}_1) \partial \bar{Z}^j(z_2) \rangle = -\frac{\bar{D}^j}{(\bar{z}_1 - z_2)^2}$, and: $\langle \bar{\Psi}^j(z_1) \widetilde{\Psi}^j(\bar{z}_2) \rangle = \frac{\bar{D}^j}{z_1 - \bar{z}_2}$, $\langle \widetilde{\Psi}^j(\bar{z}_1) \bar{\Psi}^j(z_2) \rangle = \frac{\bar{D}^j}{\bar{z}_1 - z_2}$, $\langle \widetilde{\Psi}^j(\bar{z}_1) \Psi^j(z_2) \rangle = \frac{D^j}{\bar{z}_1 - z_2}$.

of the superdiffeomorphism invariance on the string world sheet. We extract from (3.37) the kinematics $\kappa = \frac{1}{2}[(p_1 p_2)(\xi_1 \xi_2) - (p_1 \xi_2)(p_2 \xi_1)]$ following from the gauge kinetic term $\frac{1}{4}F_{\mu\nu}^{a_1}F^{a_2,\mu\nu}$ in four space-time dimensions. Here V_{CKG} is the volume of the conformal Killing group $PSL(2, \mathbf{R})$, which leaves the boundary ($\text{Im}(z) = 0$) of the disk fixed. The correlators appearing in (3.37) are of the form described in section 2. According to $Eq.$ (2.5), the closed string momentum q is split into $q^\parallel = \frac{1}{2}(q + Dq) = \frac{1}{2}(k_3 + k_4)$. The contraction of the exponentials in (3.37) yields

$$\mathcal{E} := \langle e^{ik_1 X(z_1)} e^{ik_2 X(z_2)} e^{ik_3 X(\bar{z}_3)} e^{ik_4 X(z_3)} \rangle = |z_1 - z_2|^s |z_1 - \bar{z}_3|^t |z_1 - z_3|^u \times |z_2 - \bar{z}_3|^u |z_2 - z_3|^t |\bar{z}_3 - z_3|^s . \quad (3.38)$$

The last expression is subject to the momentum constraints (2.7) and (2.10). Due to $PSL(2, \mathbf{R})$ invariance on the disk, we may fix three vertex positions. A convenient choice for the kind of setup we are considering (*cf.* [27]) is¹⁰:

$$z_1 = x \quad , \quad z_2 = -x \quad , \quad z_3 = i \quad , \quad \bar{z}_3 = -i . \quad (3.40)$$

This choice implies the c -ghost contribution:

$$\langle c(z_2) c(z_3) \tilde{c}(\bar{z}_3) \rangle = (z_2 - z_3)(z_2 - \bar{z}_3)(z_3 - \bar{z}_3) = 2i (1 + x^2) . \quad (3.41)$$

After including this correlator, we obtain:

$$\begin{aligned} \mathcal{A}_{A^{a_1} A^{a_2} T^j} &= \frac{i}{4\pi} 2^{-1-4t} \frac{\overline{D}^j}{T^j - \overline{T}^j} \left\{ (\xi_1 \xi_2) (1 + 2t) \frac{i}{4} \int_0^\infty dx \frac{(x^2 + 1)^{2t+1}}{x^{2t+2}} \right. \\ &+ \left[(k_3 \xi_1)(k_4 \xi_2) \int_0^\infty dx \frac{(x + i)^{2t+1}(x - i)^{2t-1}}{x^{2t+1}} \right. \\ &\left. \left. - (k_4 \xi_1)(k_3 \xi_2) \int_0^\infty dx \frac{(x + i)^{2t-1}(x - i)^{2t+1}}{x^{2t+1}} \right] \right\} . \end{aligned} \quad (3.42)$$

The relevant integrals may be performed with the more general formula

$$\begin{aligned} I(\delta, \alpha) &= \int_0^\infty dx x^{\delta-1} (x - i)^{\alpha-\delta} (x + i)^{-\alpha-\delta} \\ &= \sqrt{\pi} 2^{-\delta} e^{-\frac{1}{2}\pi i \alpha} \frac{\Gamma\left(\frac{\delta}{2}\right) \Gamma\left(\frac{1}{2} + \frac{\delta}{2}\right)}{\Gamma\left(\frac{1}{2} + \frac{\delta}{2} - \frac{\alpha}{2}\right) \Gamma\left(\frac{1}{2} + \frac{\delta}{2} + \frac{\alpha}{2}\right)} , \end{aligned} \quad (3.43)$$

¹⁰ With this choice the correlator (3.38) becomes

$$\mathcal{E} = \left(\frac{x^2 + 1}{4x} \right)^{2t} . \quad (3.39)$$

which will be relevant in the following. To this end, we arrive at:

$$\begin{aligned}\mathcal{A}_{A^{a_1}A^{a_2}T^j} &= \frac{1}{4} \frac{\overline{D}^j}{T^j - \overline{T}^j} \{ t (\xi_1 \xi_2) + [(k_3 \xi_1)(k_4 \xi_2) + (k_4 \xi_1)(k_3 \xi_2)] \} t \frac{\Gamma(-2t)}{\Gamma(1-t)^2} \\ &= \frac{i}{2} \frac{\overline{D}^j}{T_2^j} \left[\frac{1}{2} (p_1 p_2) (\xi_1 \xi_2) - \frac{1}{2} (p_1 \xi_2) (p_2 \xi_1) \right] t \frac{\Gamma(-2t)}{\Gamma(1-t)^2} .\end{aligned}\tag{3.44}$$

Due to $t \frac{\Gamma(-2t)}{\Gamma(1-t)^2} = -\frac{1}{2} + \mathcal{O}(k^2)$, at second order in the momenta we obtain from (3.44) the gauge kinematics κ of two gauge bosons.

To compare with the field theory, the gauge field vertices have to be rescaled as $V_{A^a} \rightarrow g_a^{-1} V_{A^a}$. This manipulation makes sure, that the string amplitude (3.37), calculated with the gauge vertices in the string basis (3.29), describes a field theoretical quantity. The leading term of the L/R symmetrized amplitude can then be equated with the T_2^j -derivative

$$\mathcal{A}_{A^{a_1}A^{a_2}T_2^j} = i(\mathcal{A}_{A^{a_1}A^{a_2}T^j} - \mathcal{A}_{A^{a_1}A^{a_2}\overline{T}^j}) = \kappa \frac{1}{4} \frac{D^j + \overline{D}^j}{T_2^j} g_a^{-2} = \kappa \partial_{T_2^j} g_a^{-2} .\tag{3.45}$$

On the other hand, the amplitudes $\mathcal{A}_{A^{a_1}A^{a_2}U^j}$ and $\mathcal{A}_{A^{a_1}A^{a_2}\overline{U}^j}$ give a vanishing result due to internal charge conservation. A result essentially following from the correlator $\langle \Psi^j(z_1) \tilde{\Psi}^j(\overline{z}_2) \rangle = 0$ in (3.34). This yields the additional differential equations:

$$\partial_{U^j} g_a^{-2} = 0 \quad , \quad \partial_{\overline{U}^j} g_a^{-2} = 0 \quad , \quad j = 1, 2, 3 .\tag{3.46}$$

The equations (3.45) and (3.46) have the solution:

$$g_a^{-2} = \text{const.} \quad e^{-\phi_4} \prod_{j=1}^3 \frac{\sqrt{(T_2^j)^2 + (f^j)^2}}{\sqrt{T_2^j}} .\tag{3.47}$$

We included the dilaton factor arising in the path integral of any string scattering on the disk. The latter has Euler number $\chi = 1$. This dilaton dependence may be explicitly checked by calculating a scattering of two gauge fields and the dilaton field ϕ_4 on the disk (*cf.* Appendix B).

Alternatively working instead with the moduli (3.30) in the real basis we obtain the differential equation:

$$\begin{aligned}\partial_{T_2^j} g_a^{-2} &= \frac{1}{4} \text{Tr} \left\{ \frac{\partial}{\partial T_2^j} (g^j + b^j) D^j \right\} g_a^{-2} \\ &= \frac{1}{4} \text{Tr} \left\{ \frac{\partial}{\partial T_2^j} (g^j + b^j) [-(g^j)^{-1} + 2 (g^j + \mathcal{F}^j)^{-1}] \right\} g_a^{-2} \quad , \quad j = 1, 2, 3 ,\end{aligned}\tag{3.48}$$

which may be cast into:

$$\partial_{T_2^j} \ln g_a^{-2} = \frac{1}{2} \partial_{T_2^j} \ln \left\{ \det [1 + (b^j + 2\pi\alpha' F^j)(g^j)^{-1}] \sqrt{\det g^j} \right\} . \quad (3.49)$$

from which one concludes:

$$g_a^{-2} = \text{const.} \ e^{-\phi_4} (\det g)^{-1/4} \sqrt{\det(g + b + 2\pi\alpha' F)} , \quad (3.50)$$

in precise agreement with (3.11) for $p = 9$. It is quite remarkable, that our disk calculation (3.37), which involves both open and closed string states and which borrows quantities (2.11) from non-commutative field theory, precisely reproduces the coupling (3.15) following from the Born-Infeld action (3.2).

3.4. Scattering of two gauge fields and one modulus from D -branes at angles

Our findings (3.45) and (3.46), describing the moduli dependence of gauge couplings of $D9$ -branes with fluxes F^j , directly translate into the T -dual picture (3.18) of $D6$ -branes at angles θ^j (*cf.* (3.21)):

$$\begin{aligned} \partial_{U_2'^j} g_a^{-2} &= -\frac{1}{4U_2'^j} \left(\frac{n^j - m^j \bar{U}^j}{n^j - m^j U^j} + \text{c.c.} \right) g_a^{-2} , \\ \partial_{T'^j} g_a^{-2} &= 0 \quad , \quad \partial_{\bar{T}^j} g_a^{-2} = 0 \quad , \quad j = 1, 2, 3 . \end{aligned} \quad (3.51)$$

These differential equations just follow from (3.45) and (3.46) after performing the duality transformation (3.18). They integrate to (3.26).

On the other hand, an explicit calculation of the string disk amplitudes $\langle A^a A^a U_2' \rangle$ and $\langle A^a A^a T_2' \rangle$ involving the dual moduli fields T'^j and U'^j should also yield the above equations. In the following we give a derivation of the CFT correlators for branes at angles. It will become obvious that the IIA computation indeed yields (3.51). In addition the discussion leads to a simple interpretation of the D matrices in the $f \neq 0$ correlators in (3.34), that have been obtained above by transforming the correlators of [32,33] to the complex basis.

The moduli on a single torus T^2 with radii R_i are defined in (3.4). In the following we drop the primes on the IIA moduli for simplicity. Let Y^a denote the two bosons corresponding to the lattice basis, $Y^a \sim Y^a + 1$. They have the correlation functions

$$\langle Y^a(z) Y^b(w) \rangle = -g^{ab} \ln |z - w|^2, \quad g^{ab} = \frac{1}{T_2 U_2} \begin{pmatrix} |U|^2 & -U_1 \\ -U_1 & 1 \end{pmatrix} .$$

The complex coordinates on T defined in (3.31)

$$Z = \sqrt{\frac{T_2}{2U_2}} (Y^1 + \bar{U} Y^2),$$

have correlators $\langle Z(z)\bar{Z}(w) \rangle = -\ln|z-w|^2$. The coordinates Z are related to any orthogonal basis X^a by

$$Z = \frac{\delta}{\sqrt{2}}(X^1 + iX^2), \quad (3.52)$$

where $\delta = e^{i\alpha}$ is a yet arbitrary phase factor.

On the double cover of the disc, the interactions between left- and right-movers at the boundary are given by $\langle \partial X^a(z) \bar{\partial} X^a(\bar{w}) \rangle = D^a \langle \partial X^a(z) \partial X^a(\bar{w}) \rangle$, where $D^a = \pm 1$ in the N and D directions, respectively. We may fix the coordinates such that X^2 is the D direction, so the $D1$ -string wraps along X^1 . The non-vanishing correlators are then:

$$\langle \partial Z(z) \bar{\partial} \bar{Z}(w) \rangle = -\frac{1}{(z-w)^2}, \quad \langle \partial Z(z) \bar{\partial} Z(\bar{w}) \rangle = -\frac{\delta^2}{(z-\bar{w})^2}, \quad (3.53)$$

and their complex conjugates. Note the non-vanishing correlator of two holomorphic coordinates on the torus, induced by the Dirichlet boundary.

We see that in the complex basis, the effect of the mixed boundary conditions on the CFT computation is quite trivial when using holomorphic fields ∂Z on the double. The computation essentially reduces to the pure Neumann case after the replacement $\bar{\partial} \bar{Z}(\bar{w}) \rightarrow \partial Z(\bar{w})$, up to a simple product of phase factors δ , collected from the vertex operators. By supersymmetry, a similar comment applies to the correlators of the fermionic superpartners Ψ .

The angle of the $D1$ -brane in the lattice basis can be identified by rewriting the X^a in terms of the Y^a . A string wrapping the class $ne_1 + me_2$ of the lattice corresponds to the ratio $F = m/n$ of the coefficients of Y^2 and Y^1 in X^1 , respectively. In this way one recovers the definitions $\tan(\alpha) = \frac{f}{U_2}$ with $f = F - U_1$. Since the above describes the correlators for a $D1$ -brane wrapped on T^2 , or, in the dual language, correlators in the presence of non-zero B_{NS} fields, Eq. (3.53) must be simply the non-commutative correlators of [32,33], written in the natural complex basis on the torus. This fact is verified by (3.34), using $\bar{D} = \delta^2$ and noting that the T -duality transformation $X_R^2 \rightarrow -X_R^2$ implies the replacement $\bar{\partial} Z$ by $\bar{\partial} \bar{Z}$ when going from IIA to IIB . With these replacements, the IIB vertex operators $V_{T/U}$ and their amplitudes immediately translate to the T -dual IIA vertex operators $V_{U'/T'}$ and their amplitudes, and eventually lead to the T -dual equations (3.51). In the following we will sometimes use this simple direct relation between IIA and IIB amplitudes to switch between the two T -dual descriptions.

4. Untwisted matter metric

In this section, we consider tree-level disk scattering amplitudes involving matter fields inserted at the boundary of the disk and the bulk moduli fields. We shall determine the metric $G_{C_i \overline{C}_k}$ for untwisted charged matter fields C_i and \overline{C}_k . In order to obtain a chiral $N=1$ theory in $D = 4$ type *IIB* or type *I*, the T^6 -torus is modded out by some orbifold group. The details of the latter are not relevant for untwisted matter fields originating from the untwisted sector. Generically in type *IIB* or type *I* there appear untwisted charged matter fields C_i^9 , $i = 1, 2, 3$ from open strings starting and ending on $D9$ -branes. They are charged under the gauge group resulting from the $D9$ -branes. Similarly, open strings starting and ending on the same $D5_j$ -brane give rise to the matter fields $C_i^{5_j}$, $i, j = 1, 2, 3$. Here the index j denotes the torus $T^{2,j}$, on which the $D5$ is wrapped. The open strings are only charged w.r.t. the gauge group from the $D5_j$.

Before we start let us add the following comment: in the heterotic string the corresponding 3-point amplitudes on the sphere between two matter fields and one modulus vanish due to internal charge conservation. Therefore in the heterotic string one has to compute 4-point amplitudes between two matter fields and two moduli in order to extract non-trivial informations about the heterotic matter field Kähler metrics. This situation now changes in type *II* models with closed and open string fields. Specifically, as we will now see, some 3-point amplitudes between two open string matter fields and one closed string modulus are allowed by internal charge conservation. This essentially stems from the fact that the closed string vertex operator can be treated as two open string vertex operators, *i.e.* one is basically computing a 4-point amplitude. However one has to be quite careful, since there can be in general additional moduli dependences for the matter field Kähler metrics that are not captured by the 3-point amplitudes. To obtain these further informations we will also calculate 4-point amplitudes with two moduli and two matter fields.

4.1. Three-point amplitudes

As in the case of the gauge couplings we first work on the type *IIB* side, where we compute the scattering of two matter fields C_i, \overline{C}_k and the closed string modulus T^j . The open string σ -model action has the boundary term $\int dz A_i \partial X^i$, which may be written in complex coordinates (3.31) as

$$\sum_{i=1}^6 A_i \partial X^i = \sum_{j=1}^3 \frac{\sqrt{2}}{(U^j - \overline{U}^j)^{1/2} (T^j - \overline{T}^j)^{1/2}} \left[C_j \partial Z^j - \overline{C}_j \partial \overline{Z}^j \right], \quad (4.1)$$

with $C_i = A_{2i-1} + U^i A_{2i}$, $i = 1, 2, 3$. From this term the vertex operator for untwisted matter fields C_i in the zero ghost picture may be read off¹¹:

$$V_{C_i}^{(0)}(z, k) = \lambda \left[\partial Z^i + i(k\psi)\Psi^i \right] e^{ik_\nu X^\nu(z)} . \quad (4.2)$$

The latter is inserted at the boundary of the disk. This vertex operator involves untwisted fields from the internal torus directions inserted at the boundary.

We must not consider couplings of matter fields C_i and \overline{C}_k referring to different internal complex planes i and k . Due to internal charge conservation those couplings must vanish at second order in the matter fields:

$$G_{C_i \overline{C}_k} = 0 \quad , \quad i \neq k . \quad (4.3)$$

This can be easily anticipated from the form of the vertex operator (4.2). Hence we shall consider the following three-point amplitude¹²

$$\mathcal{A}_{C_i \overline{C}_i T^j} = \frac{i}{2\pi} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \langle V_{C_i}^{(0)}(z_1, k_1) V_{\overline{C}_i}^{(0)}(z_2, k_2) V_{T^j}^{(-1, -1)}(\overline{z}_3, z_3, q) \rangle , \quad (4.4)$$

with the closed string vertex operator (3.30) in the $(-1, -1)$ ghost picture. The derivative w.r.t. the physical scalar $\text{Im}(T)$ is given by the sum of (4.4) and the same amplitude with T replaced by \overline{T} .

There are two contributions to the correlator (4.4): one, denoted by X_1 , from contracting the first term of both matter field vertices (4.2) and the other, X_2 , from contracting the second terms of both matter vertices with V_{T^j} . The correlators to be done are basic and are given in (2.1) and (2.2). To this end we obtain

$$\begin{aligned} X_1 &= \frac{\overline{D}^j}{T^j - \overline{T}^j} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \mathcal{E} \frac{-1}{(\overline{z}_3 - z_3)^2 (z_1 - z_2)^2} , \\ X_2 &= \frac{\overline{D}^j}{T^j - \overline{T}^j} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \mathcal{E} \frac{1}{\overline{z}_3 - z_3} \frac{k_1 k_2}{z_1 - z_2} \left(\frac{1}{(z_1 - z_2)(\overline{z}_3 - z_3)} + \frac{\delta^{ij}}{(z_1 - z_3)(z_2 - \overline{z}_3)} \right) \end{aligned} \quad (4.5)$$

¹¹ We discard the normalization $\frac{\sqrt{2}}{(U - \overline{U})^{1/2} (T - \overline{T})^{1/2}}$, in order for the string vertex to yield canonical normalized kinetic energy in the string basis.

¹² Compared to (3.37) we disregarded a factor of $\frac{1}{2}$, which takes into account the factor $\sqrt{2}$ in (4.1).

with the requirement $s = -2t$. As in subsection 2.3, we fix three vertex positions according to (3.40) and introduce the correlator (3.41) to arrive at

$$\begin{aligned}
X_1 &= -\frac{1}{\pi} 2^{-4-4t} I(-1-2t, 0) \frac{\overline{D}^j}{T^j - \overline{T}^j} = -\frac{1}{\sqrt{\pi}} 2^{-2t-3} \frac{\Gamma(-\frac{1}{2}-t)}{\Gamma(-t)} \frac{\overline{D}^j}{T^j - \overline{T}^j}, \\
X_2 &= \frac{i}{\pi} 2^{-1-4t} t \left[\frac{i}{4} I(-1-2t, 0) - \delta^{ij} I(-2t, 1) \right] \frac{\overline{D}^j}{T^j - \overline{T}^j} \\
&= -\frac{2^{-2-2t}}{\sqrt{\pi}} [t - \delta^{ij}(1+2t)] \frac{\Gamma(-\frac{1}{2}-t)}{\Gamma(-t)} \frac{\overline{D}^j}{T^j - \overline{T}^j},
\end{aligned} \tag{4.6}$$

with the integral $I(\delta, \alpha)$ introduced in (3.43). Hence the final result for the amplitude (4.4) is:

$$\mathcal{A}_{C_i \overline{C}_i T^j} = \frac{it}{\sqrt{\pi}} (1 - 2\delta^{ij}) \frac{\overline{D}^j}{T_2^j} 2^{-3-2t} \frac{\Gamma(\frac{1}{2}-t)}{\Gamma(1-t)}. \tag{4.7}$$

For low momenta $t = -2p_1 p_2$, the result agrees with the D -brane effective action for the scalar matter fields coupling to the closed string modulus T^j . Therefore, the first term of the expansion

$$\mathcal{A}_{C_i \overline{C}_i T^j} = -i (p_1 p_2) \frac{\overline{D}^j}{4T_2^j} (1 - 2\delta^{ij}) + \mathcal{O}(t^2) \tag{4.8}$$

describes the derivative of the metric $G_{C_i \overline{C}_i}$ for the scalar matter fields C_i and \overline{C}_i . Up to a minus sign we obtain the same result for amplitude with an insertion of the conjugate vertex operator for \overline{T}^j , leading to the differential equation

$$\partial_{T_2^j} G_{C_i \overline{C}_i} = i(\mathcal{A}_{C_i \overline{C}_i T^j} - \mathcal{A}_{C_i \overline{C}_i \overline{T}^j}) = \frac{D^j + \overline{D}^j}{4T_2^j} (1 - 2\delta^{ij}) G_{C_i \overline{C}_i}. \tag{4.9}$$

We have changed the normalization of the matter field vertex operator (4.2). The latter refers to the string basis and leads to canonically normalized matter metrics $G_{C_i \overline{C}_i} \sim 1$. In order that our string amplitude reproduces a field-theory result, we have to normalize it properly. This requires the redefinition: $V_{C_i} \rightarrow G_{C_i \overline{C}_i}^{1/2} V_{C_i}$.

For convenience, we shall discuss first the solution with the total internal gauge flux $f^j \equiv \mathcal{F}^j$ turned off, *i.e.* $b^j = -2\pi F^j$. According to (3.20), this means, that in the case of vanishing fluxes, on the dual type IIA side, the $D6$ -branes become parallel $\theta^j = 0$. From *Eq.* (3.36), in the case without fluxes $f^j \rightarrow 0$, the mixing parameter becomes ± 1 , depending on the boundary conditions of the open string w.r.t. the plane j . In that case the system (4.9) simplifies.

Let us first consider the matter fields C_i^9 from the $D9$ -brane without fluxes. In that case, all open string coordinates obey Neumann boundary conditions. Therefore we have $D^j = +1$ and find:

$$G_{C_i^9 \overline{C}_i^9} = \alpha'^{-3/2} e^{-\phi_4} \sqrt{\frac{(T^k - \overline{T}^k)(T^l - \overline{T}^l)}{T^i - \overline{T}^i}} \quad , \quad (i, k, l) = \overline{(1, 2, 3)} \quad . \quad (4.10)$$

Now let us move on to the charged matter fields $C_i^{5,k}$ arising from a $D5$ -brane, which is wrapped around the torus $T^{2,k}$. All open string coordinates orthogonal to the $D5$ -brane obey Dirichlet boundary conditions, *i.e.* $D^j = -1$ w.r.t. the two transverse tori $T^{2,j}$. On the other hand, we have $D^k = +1$ w.r.t. $T^{2,k}$. To be more concrete, let us discuss the case $k = 1$, *i.e.* the $D5$ -brane is wrapped around the torus $T^{2,1}$. In that case, the equations (4.9) are solved by:

$$G_{C_i^{5,1} \overline{C}_i^{5,1}} = e^{-\phi_4} \times \begin{cases} \alpha'^{1/2} \frac{1}{\sqrt{(T^1 - \overline{T}^1)(T^2 - \overline{T}^2)(T^3 - \overline{T}^3)}} \quad , \quad i = 1 \quad , \\ \alpha'^{-3/2} \sqrt{\frac{(T^1 - \overline{T}^1)(T^2 - \overline{T}^2)}{T^3 - \overline{T}^3}} \quad , \quad i = 2 \quad , \\ \alpha'^{-3/2} \sqrt{\frac{(T^1 - \overline{T}^1)(T^3 - \overline{T}^3)}{T^2 - \overline{T}^2}} \quad , \quad i = 3 \quad . \end{cases} \quad (4.11)$$

The other cases $k = 2, 3$ may be obtained from (4.11) by permutation of the planes. So far, we have written our metrics in terms of the string moduli introduced in (3.4) to which the string vertex operators (3.30) refer. To obtain the metrics (4.10) and (4.11) in the field theory basis, we only have to replace the T^j -moduli through the field theory moduli s and t^j , introduced in (3.12) and (3.13). This leads to

$$G_{C_i^9 \overline{C}_i^9} = \frac{\kappa_4^{-2}}{t^i + \overline{t}^i} \quad , \quad G_{C_i^{5,1} \overline{C}_i^{5,1}} \sim \begin{cases} \frac{\kappa_4^{-2}}{s + \overline{s}} \quad , \quad i = 1 \quad , \\ \frac{\kappa_4^{-2}}{t^3 + \overline{t}^3} \quad , \quad i = 2 \quad , \\ \frac{\kappa_4^{-2}}{t^2 + \overline{t}^2} \quad , \quad i = 3 \quad . \end{cases} \quad (4.12)$$

In the following we allow for non-vanishing fluxes f^j on the $D9$ -brane. With the same arguments as in the case of gauge couplings, we shall stick to the $D9$ -case only. The relevant calculation has been already performed in the previous subsection. We only have to take into account the flux dependence of D^j , given in (3.35) and integrate the differential equations (4.9). The solution is:

$$G_{C_i \overline{C}_i} = \text{const.} \quad e^{-\phi_4} \sqrt{\frac{T^i - \overline{T}^i}{(T^k - \overline{T}^k)(T^l - \overline{T}^l)}} \frac{|T_2^k + i f^k| |T_2^l + i f^l|}{|T_2^i + i f^i|} \quad , \quad (4.13)$$

for $(i, k, l) = \overline{(1, 2, 3)}$. It is evident, that this expression boils down to (4.10) in the case of vanishing fluxes $f^j = b^j + 2\pi\alpha' F^j = 0$. In the case of N=1 supersymmetry, we may borrow Eq. (3.16) to cast (4.13) into

$$G_{C_i \overline{C}_i} = \alpha'^{-3/2} e^{-\phi_4} \left| 1 - \tilde{f}^k \tilde{f}^l \right| \sqrt{\frac{(T^k - \overline{T}^k)(T^l - \overline{T}^l)}{T^i - \overline{T}^i}} \quad , \quad (i, k, l) = \overline{(1, 2, 3)} \quad , \quad (4.14)$$

where $\tilde{f}^i = f^i/T_2^i$. This should be compared with (4.10). Finally, written in terms of the field-theory moduli s and t^j , we obtain:

$$G_{C_i \overline{C}_i} = \kappa_4^{-2} \left| 1 - \tilde{f}^k \tilde{f}^l \right| \frac{1}{t^i + \overline{t}^i} \quad , \quad (i, k, l) = \overline{(1, 2, 3)} \quad . \quad (4.15)$$

Note, that \tilde{f}^i is a dimensionless quantity. Through (3.18) the metric (4.15) translates to the type IIA side with intersecting branes into:

$$G_{C_i \overline{C}_i} = \kappa_4^{-2} \left| 1 - \tilde{f}^k \tilde{f}^l \right| \frac{1}{u^i + \overline{u}^i} \quad , \quad (i, k, l) = \overline{(1, 2, 3)} \quad , \quad (4.16)$$

where now $\tilde{f}^i = \tan(\pi\theta^i)$.

4.2. Four-point amplitudes

By internal charge conservation, the type IIB three-point function with two matter fields and one U modulus vanishes and we instead proceed by factorizing a four-point function. The relation between the $\alpha' \rightarrow 0$ limit of the string four-point function and the supersymmetric effective theory has been studied in [2,41] for the heterotic string. Although some details are different on the type I string side, the computation on the field theory side is the same and we refer to the clear exposition in [2] for details.

The $\alpha' \rightarrow 0$ limit of the string correlation function $\mathcal{A} = \langle C^a M^b \overline{M}^c \overline{C}^d \rangle$ with two matter fields C and two moduli fields M is [2]

$$\mathcal{A} \sim G_{C^a \overline{C}^d} G_{M^b \overline{M}^c} \frac{ts}{u} + s R_{C^a \overline{C}^d M^b \overline{M}^c} + \mathcal{A}_{pot} + \mathcal{O}(k^4) \quad , \quad (4.17)$$

where R is the Riemann curvature of the Kähler manifold

$$R_{C^a \overline{C}^d M^b \overline{M}^c} = K_{C^a M^b \overline{M}^c \overline{C}^d} - K_{C^a M^b \overline{C}^e} G^{\overline{C}^e C^f} K_{\overline{C}^d \overline{M}^c C^f}$$

and the kinematic invariants are defined in (2.9). The k_i are the momenta of the external fields in \mathcal{A} , and $G_{C\overline{C}} = K_{C\overline{C}} \equiv \partial_C \partial_{\overline{C}} K$, with K the Kähler potential. The first term in Eq. (4.17) is due to graviton exchange, the second term from the sigma model couplings arising from the Kähler potential K and the term \mathcal{A}_{pot} denotes couplings due to the scalar

potential from F - and D -terms. The first two terms in (4.17) can be used to determine the moduli dependence of the Kähler potential for the matter fields by integration.

For the present case of a compactification on the factorized tori $T^{2,i}$, there are some significant simplifications. The metric for the untwisted moduli fields is diagonal, $G_{M^a \overline{M}^b} = \delta^{ab} (M^a + \overline{M}^a)^{-2}$. For M a complex structure modulus, $M = u^i$, the contribution \mathcal{A}_{pot} vanishes and moreover the u dependence of the matter metric factorizes

$$G_{C^a \overline{C}^b} = \delta_{ab} \tilde{G}_a \prod_i H_a^i(u^i). \quad (4.18)$$

Here \tilde{G} contains the dependence on all other moduli but the u^i . For quadratic Kähler potential the metric can be always diagonalized as above and we will sometimes suppress the index a for simplicity. For further convenience we note, that for a simple dependence $H(u) = (u + \overline{u})^{-q}$, the ratio of the coefficients of the graviton and the sigma model channels in (4.17) is the exponent q .

The U moduli dependence of the matter metric may be extracted from the 4-point function $\mathcal{A} = \langle V_{C^a} V_{u^m} V_{\overline{u}^n} V_{\overline{C}^b} \rangle$,

$$\mathcal{A} \sim \prod \int dx_i \mathcal{J}(x_i) \langle V_{C^a}^{-1}(x_1, k_1) V_{u^m}^0(x_2, k_2; x_3, k_3) V_{\overline{u}^n}^0(x_4, k_4; x_5, k_5) V_{\overline{C}^b}^{-1}(x_6, k_6) \rangle,$$

where $\mathcal{J}(x_i)$ is the Jacobian for fixing of the $PSL(2, \mathbf{R})$ symmetry.

The only non-trivial point is the evaluation of the integrals over three real positions. We proceed by choosing a singular gauge choice for $PSL(2, \mathbf{R})$, where all bulk coordinates are proportional to a scale factor $w \in \mathbf{R}$, $x_i = w y_i$, $i = 2, \dots, 5$. The Jacobian for the coordinate transformation is singular at $w = 0, \infty$ and the integrals need to be regularized by cutting small discs around these values. The w -integral will eventually decouple. A particularly convenient choice is

$$x_1 = \infty, \quad x_2 = w y, \quad x_3 = w \overline{y}, \quad x_4 = w, \quad x_5 = w, \quad x_6 = 0. \quad (4.19)$$

At $w = 0, \infty$ the bulk operators collide with a boundary operator. Note that the bulk operators have no singularity as they approach a generic point on the boundary and in fact the special choice (4.19) places the second bulk operator at the boundary.

It is now straightforward to evaluate the integrand, and we find

$$\begin{aligned} \mathcal{A} &\sim \int \frac{dw}{w} \int d^2 y |1 - y|^{u-4} |y|^t \left[2 r_1 (1 - u/2) - \frac{1}{2} r_2 u \left(\frac{1 - \overline{y}}{\overline{y}} + \frac{1 - y}{y} \right) \right] \\ &\sim r_1 \frac{ts}{u} + r_2 s + \mathcal{O}(k^4), \end{aligned} \quad (4.20)$$

where $r_1 = \delta_{ab} \delta_{mn}$ and $r_2 = r_1 \delta_{am}$. Note that the w -integral decouples, as promised. The y integral has the standard representation in terms of Gamma functions

$$\int d^2y y^a \bar{y}^{a'} (1-y)^b (1-\bar{y})^{b'} = \pi \frac{\Gamma(1+a)\Gamma(1+b)}{\Gamma(2+a+b)} \frac{\Gamma(-1-a'-b')}{\Gamma(-a')\Gamma(-b')},$$

(provided $a - a' \in \mathbf{Z}$, $b - b' \in \mathbf{Z}$) and leads to the small momentum expansion quoted above.

As noted below (4.18) the relative coefficient of the two terms at $\mathcal{O}(k^2)$ determine the u dependence of $G_{C^a \bar{C}^a}$ to be

$$\prod_i H_a^i(u^i) = (u^a + \bar{u}^a)^{-1}. \quad (4.21)$$

Let us summarize at this point all the above results in terms of the Kähler potential for the untwisted type IIB moduli and to second order in the matter fields:

$$\begin{aligned} \kappa_4^2 K = & -\ln(s + \bar{s}) - \sum_{i=1}^3 \ln(t^i + \bar{t}^i) - \sum_{i=1}^3 \ln(u^i + \bar{u}^i) \\ & + \sum_{i=1}^3 \frac{|C_i^9|^2}{(u^i + \bar{u}^i)(t^i + \bar{t}^i)} \sqrt{\frac{(1 + (\tilde{f}^k)^2)(1 + (\tilde{f}^l)^2)}{1 + (\tilde{f}^i)^2}} \\ & + \sum_{i=1}^3 \frac{|C_i^{5,i}|^2}{(s + \bar{s})(u^i + \bar{u}^i)} + \sum_{i,j,k}^3 \frac{d_{ijk} |C_j^{5,k}|^2}{(t^i + \bar{t}^i)(u^j + \bar{u}^j)}. \end{aligned} \quad (4.22)$$

Here we have introduced the tensor d_{ijk} which is 1 for (i, j, k) a permutation of $(1, 2, 3)$ and 0 otherwise. Note that (4.22) is written in the Einstein frame, which accounts for an extra factor $e^{2\phi_4}$ in the metric. In the case of fixed complex structure and zero fluxes \tilde{f} , the above Kähler potential agrees with the results of [34], obtained there by T -duality arguments.

5. Twisted matter metric

5.1. Twist fields and twist field correlators

In the following we will work in type IIA . Here, in intersecting brane world models, the chiral matter comes from open strings stretched between two intersecting branes. Hence these open strings obey mixed boundary conditions [7]:

$$\begin{aligned} \text{Re } \partial_\sigma Z^j \Big|_{\sigma=0} & , \quad \text{Im } Z^j \Big|_{\sigma=0} = 0 , \\ \text{Re } e^{i\theta^j} \partial_\sigma Z^j \Big|_{\sigma=\pi} & , \quad \text{Im } e^{i\theta^j} Z^j \Big|_{\sigma=\pi} = 0 , \end{aligned} \quad (5.1)$$

with θ^j being the relative angle of the two branes w.r.t. the j -th internal torus $T^{2,j}$. These boundary conditions produce cuts $\sim z^{-\theta^j}$ in the map $z \rightarrow Z^j(z)$ from the disk to the target space. In the vertex operator for the corresponding scalar matter field C_θ , these cuts are introduced¹³ on the disk by the twist fields $\sigma_{\theta^j}(z)$. The world sheet supercurrent on the boundary ($z = \bar{z}$) of a disk is given by:

$$T_F(z, \bar{z}) = \frac{1}{2} [T_F(z) + \tilde{T}_F(\bar{z})] ,$$

$$T_F(z) = \partial X_\mu(z) \psi^\mu(z) + \sum_{i=1}^3 \partial \bar{Z}^i(z) \Psi^i(z) + \partial Z^i(z) \bar{\Psi}^i(z) ,$$

$$\tilde{T}_F(\bar{z}) = \bar{\partial} X_\mu(\bar{z}) \tilde{\psi}^\mu(\bar{z}) + \sum_{i=1}^3 \bar{\partial} \bar{Z}^i(\bar{z}) \tilde{\Psi}^i(\bar{z}) + \bar{\partial} Z^i(\bar{z}) \bar{\tilde{\Psi}}^i(\bar{z}) .$$
(5.2)

The latter is only invariant under the twist if the internal fermions $\Psi, \tilde{\Psi}$, which may be bosonized according to $\Psi^j(z) = e^{iH^j(z)}, \tilde{\Psi}^j(\bar{z}) = e^{i\tilde{H}^j(\bar{z})}$ are twisted as well. This is accomplished by the fermionic twist fields $s_{\pm\theta^j}(z)$, which are bosonized $s_{\pm\theta^j}(z) = e^{\pm i(1-\theta^j)H^j(z)}$. Here, the fields $H^j(z)$ correspond to the internal bosonized world-sheet fermion ($j = 1, 2, 3$).

Generically two intersecting stacks a and b (of N and M $D6$ branes, respectively) establish¹⁴ the gauge group $U(N) \times U(M)$. The massless (twisted) R -sector of open strings stretched between these two stacks a and b intersecting at an angle θ gives rise to massless chiral fermions in the bifundamental (N, \bar{M}) of $U(N) \times U(M)$. In the case of $N=1$ supersymmetry these fermions build chiral $N=1$ multiplets whose scalars (scalar matter fields C_θ, \bar{C}_θ) stem from the open strings in the (twisted) NS -sector. Hence these chiral multiplets sit at the intersection of the two stacks a and b and their vertex operators have to be inserted at the boundary of any string world sheet diagram. The vertex operator for an $N=1$ matter field C_θ inserted at the boundary $z = \bar{z}$ of the disk is (cf. e.g. [7])

$$V_{C_\theta}^{(-1)}(z, k) = \lambda e^{-\phi(z)} \prod_{j=1}^3 s_{\theta^j}(z) \sigma_{-\theta^j}(z) e^{ik_\nu X^\nu(z)} \quad (5.3)$$

in the -1 ghost picture. Here, the twist fields $s_{\theta^j}, \sigma_{\theta^j}$ depend on the angles $\theta = (\theta^1, \theta^2, \theta^3)$ between the two intersecting branes describing the details of the chiral matter under consideration. The bosonic twist fields σ_{θ^j} are responsible for twisting the NS ground state, while the twisted spin fields s_{θ^j} are required by supersymmetry (see the argument after

¹³ This is notorious from heterotic orbifold compactifications [42].

¹⁴ In the case, when both stacks do not sit on top of the orientifold planes. Otherwise, if one stack is placed at an orientifold plane the gauge group is $SO(2N)$ or $USp(N)$.

Eq. (5.2)). The Chan–Paton factor λ describes the gauge degrees of freedom at both string ends. Similarly, for the conjugate matter field \overline{C}_θ , originating from the same open string, but with different orientation, we have:

$$V_{\overline{C}_\theta}^{(-1)}(z, k) = \lambda^\dagger e^{-\phi(z)} \prod_{j=1}^3 s_{-\theta^j}(z) \sigma_{\theta^j}(z) e^{ik_\nu X^\nu(z)}. \quad (5.4)$$

The techniques of correlation functions with twist field operators σ_{θ^j} on the sphere have been pioneered in [42,43] and generalized in [44–47]. These results may be borrowed to also determine twist field correlation functions on the disk [48,49]. Correlators with twist fields located on the boundary $z = \bar{z}$ of a disk are essentially obtained¹⁵ from the corresponding results on the sphere by “taking the square root”. This procedure leads to the following basic correlators on the disk (for one complex twisted dimension):

$$\begin{aligned} \langle \sigma_\theta(z_1) \sigma_{-\theta}(z_2) \rangle &= (z_1 - z_2)^{-\theta(1-\theta)}, \\ \langle s_\theta(z_1) s_{-\theta}(z_2) \rangle &= (z_1 - z_2)^{-(1-\theta)^2}. \end{aligned} \quad (5.5)$$

The fields $\sigma_\theta(z)$ and $s_\theta(z)$ have conformal dimensions $h_{\sigma_\theta} = \frac{1}{2}\theta(1-\theta)$ and $h_{s_\theta} = \frac{1}{2}(1-\theta)^2$, respectively. With this information it is easy to realize that the vertex operator $V_{C_\theta}^{(-1)}(z, k)$ has conformal weight $h = 1$, provided one uses the supersymmetry relation in the form $\sum_i \theta^i = 2$. The twist fields generate branchings on the complex fields $\partial Z(z), \bar{\partial} Z(\bar{z}), \Psi(z), \tilde{\Psi}(\bar{z})$, introduced in (3.31). The local behavior of those fields in the presence of twist fields is given by the operator products [42,50]

$$\begin{aligned} \partial Z^j(z) \sigma_{\theta^j}(w) &= (z-w)^{\theta^j-1} \tau_{\theta^j}(w) + \dots, \quad \partial Z^j(z) \sigma_{-\theta^j}(w) = (z-w)^{-\theta^j} \tilde{\tau}_{-\theta^j}(w) + \dots, \\ \partial \bar{Z}^j(z) \sigma_{\theta^j}(w) &= (z-w)^{-\theta^j} \tilde{\tau}_{\theta^j}(w) + \dots, \quad \partial \bar{Z}^j(z) \sigma_{-\theta^j}(w) = (z-w)^{\theta^j-1} \tau_{-\theta^j}(w) + \dots, \\ \bar{\partial} Z^j(\bar{z}) \sigma_{\theta^j}(w) &= -(\bar{z}-w)^{\theta^j-1} \tau_{\theta^j}(w) + \dots, \quad \bar{\partial} Z^j(\bar{z}) \sigma_{-\theta^j}(w) = -(\bar{z}-w)^{-\theta^j} \tilde{\tau}_{-\theta^j}(w) + \dots, \\ \bar{\partial} \bar{Z}^j(\bar{z}) \sigma_{\theta^j}(w) &= -(\bar{z}-w)^{-\theta^j} \tilde{\tau}_{\theta^j}(w) + \dots, \quad \bar{\partial} \bar{Z}^j(\bar{z}) \sigma_{-\theta^j}(w) = -(\bar{z}-w)^{\theta^j-1} \tau_{-\theta^j}(w) + \dots. \end{aligned} \quad (5.6)$$

Obviously, the OPEs involving negative angles θ^j are obtained from the ones with positive angles θ^j after performing the replacement $\theta^j \rightarrow 1 - \theta^j$. In addition, we have:

$$\begin{aligned} \Psi^j(z) s_{\theta^j}(w) &= (z-w)^{1-\theta^j} \tilde{t}_{\theta^j}(z) + \dots, \\ \Psi^j(z) s_{-\theta^j}(w) &= (z-w)^{\theta^j-1} t_{-\theta^j}(z) + \dots, \\ \bar{\Psi}^j(z) s_{\theta^j}(w) &= (z-w)^{\theta^j-1} t_{\theta^j}(z) + \dots. \end{aligned} \quad (5.7)$$

¹⁵ This is not true for scattering processes involving both fields from the boundary *and* closed strings from the bulk.

After applying the picture changing operator $P(w, \bar{w}) = e^{\phi(w)} T_F(w) + e^{\tilde{\phi}(\bar{w})} \tilde{T}_F(\bar{w})$, we obtain the matter field vertex in the zero ghost picture:

$$V_{C_\theta}^{(0)}(z, k) = \lambda \sum_{l=1}^3 \left[t_{\theta_l}(z) \tau_{-\theta_l}(z) + \frac{1}{3} i (k_\mu \psi^\mu) s_{\theta_l}(z) \sigma_{-\theta_l}(z) \right] \prod_{\substack{j=1 \\ j \neq l}}^3 s_{\theta_j}(z) \sigma_{-\theta_j}(z) e^{ik_\nu X^\nu(z)}. \quad (5.8)$$

Here, $\tau_{\theta_j}(z)$ are excited bosonic twist fields of conformal dimension $h_{\tau_\theta} = \frac{1}{2}(1-\theta)(2+\theta)$. On the other hand, the excited fermionic twist fields $t_{\theta_j}(z)$, which may be bosonized according to $t_{\pm\theta_j}(z) = e^{\mp i\theta_j H^j(z)}$, have conformal dimension $h_{t_\theta} = \frac{1}{2}\theta^2$. With this information, the operator (5.8) has conformal weight $h = 1$ after imposing the condition (3.25). In the supersymmetric case, the matter field vertex operator represents a marginal deformation of the underlying N=2 superconformal field theory. This is why it has to carry conformal weight $h = 1$ in that case and the matter field is a massless scalar. In fact, in the supersymmetric case, the scalar matter field vertex $V_{C_\theta}^{(0)}$ is the highest component of an N=2 superconformal multiplet, whose fermionic component is represented by the fermionic vertex operator.

From the local behavior of the twist fields, given in (5.6), we may derive the following correlators, which we shall need later:

$$\begin{aligned} \langle \bar{\partial} \bar{Z}^j(z) \partial Z^j(w) \sigma_{\theta_j}(x_1) \sigma_{-\theta_j}(x_2) \rangle &= - \frac{(x_1 - x_2)^{-\theta^j(1-\theta^j)}}{(z - w)^2} \frac{(w - x_1)^{\theta^j-1} (z - x_2)^{\theta^j-1}}{(w - x_2)^{\theta^j} (z - x_1)^{\theta^j}}, \\ &\quad \times [(z - x_2)(w - x_1) + \theta^j(z - w)(x_1 - x_2)], \\ \langle \bar{\partial} Z^j(\bar{z}) \partial \bar{Z}^j(w) \sigma_{\theta_j}(x_1) \sigma_{-\theta_j}(x_2) \rangle &= - \bar{D}^j \frac{(x_1 - x_2)^{-\theta^j(1-\theta^j)}}{(\bar{z} - w)^2} \frac{(\bar{z} - x_1)^{\theta^j-1} (w - x_2)^{\theta^j-1}}{(\bar{z} - x_2)^{\theta^j} (w - x_1)^{\theta^j}}, \\ &\quad \times [(w - x_2)(\bar{z} - x_1) + \theta^j(w - \bar{z})(x_1 - x_2)], \\ \langle \partial \bar{Z}^j(\bar{z}) \partial Z^j(w) \sigma_{\theta_j}(x_1) \sigma_{-\theta_j}(x_2) \rangle &= - D^j \frac{(x_1 - x_2)^{-\theta^j(1-\theta^j)}}{(\bar{z} - w)^2} \frac{(w - x_1)^{\theta^j-1} (\bar{z} - x_2)^{\theta^j-1}}{(w - x_2)^{\theta^j} (\bar{z} - x_1)^{\theta^j}}, \\ &\quad \times [(w - x_1)(\bar{z} - x_2) + \theta^j(\bar{z} - w)(x_1 - x_2)]. \end{aligned} \quad (5.9)$$

The contraction of internal fermionic fields with spin fields referring to the same complex plane becomes on the disk:

$$\begin{aligned} \langle \tilde{\Psi}^j(\bar{z}) \bar{\Psi}^j(w) s_{\theta_j}(x_1) s_{-\theta_j}(x_2) \rangle &= \frac{\bar{D}^j}{\bar{z} - w} (x_1 - x_2)^{-(1-\theta^j)^2} \left[\frac{(\bar{z} - x_1)(w - x_2)}{(\bar{z} - x_2)(w - x_1)} \right]^{1-\theta^j}, \\ \langle \bar{\tilde{\Psi}}^j(\bar{z}) \Psi^j(w) s_{\theta_j}(x_1) s_{-\theta_j}(x_2) \rangle &= \frac{D^j}{\bar{z} - w} (x_1 - x_2)^{-(1-\theta^j)^2} \left[\frac{(\bar{z} - x_2)(w - x_1)}{(\bar{z} - x_1)(w - x_2)} \right]^{1-\theta^j}. \end{aligned} \quad (5.10)$$

Finally, the two-point correlators of excited twist on the disk fields may be obtained by taking the square root of the closed string results [42] (*cf.* also [50]):

$$\begin{aligned}\langle \tau_\theta(z_1) \tau_{-\theta}(z_2) \rangle &= \theta (z_1 - z_2)^{-\theta(3-\theta)} , \\ \langle t_\theta(z_1) t_{-\theta}(z_2) \rangle &= (z_1 - z_2)^{-\theta^2} .\end{aligned}\tag{5.11}$$

On the other hand, these correlators may be also derived from *Eqs.* (5.6) and (5.9).

5.2. String computation of moduli dependence

Let us now calculate the string S -matrix¹⁶ with

$$\mathcal{A}_{C_\theta \overline{C}_\theta U^j} = \frac{i}{2\pi} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \langle V_{C_\theta}^{(-1)}(z_1, k_1) V_{\overline{C}_\theta}^{(-1)}(z_2, k_2) V_{U^j}^{(0,0)}(\overline{z}_3, z_3, q) \rangle \tag{5.12}$$

on the disk to extract the information on the metric $G_{C_\theta \overline{C}_\theta}$ of the two matter fields $C_\theta, \overline{C}_\theta$. The two matter field vertices (5.3) and (5.4) are inserted at the boundary of the disk, *i.e.* $z_1 = \overline{z}_1, z_2 = \overline{z}_2$.

The correlation functions such as (5.9) are in fact the standard correlators for the Neumann boundary conditions and not the right ones for doing the computation for branes at angles. However we know already from the discussion in section 3.4. that there is a simple way to adapt to this case: it suffices to complex conjugate the right-moving fields. Thus we write the closed string vertex operator for the complex structure modulus U^j as

$$V_{U^j}^{(0,0)}(\overline{z}, z, k) = \frac{1}{U^j - \overline{U}^j} \left[\overline{\partial} Z^j + i(k\tilde{\psi}) \widetilde{\Psi}^j(\overline{z}) \right] \left[\partial \overline{Z}^j + i(k\psi) \overline{\Psi}^j(z) \right] e^{iq_\nu X^\nu(\overline{z}, z)}, \tag{5.13}$$

and subsequently use the correlators above. To compute the correlation function of the physical scalar we will again consider the sum of the above amplitude and the one with the U operator replaced by the complex conjugate at the end.

There are two non-vanishing contributions to the amplitude (5.12). They arise from the contraction of the matter vertices with either both bosons or both fermions of the U^j -vertex operator. We shall denote these two possibilities by X_1 and X_2 , respectively. With the vertex operator (5.13), the relevant correlators accounting for the internal sector are given in (5.9) and (5.10). The correlator for the exponentials is given by the correlator \mathcal{E} introduced in (3.38). The contractions of the space-time fields are basic and given in (2.1). After putting all contributions together and using the supersymmetry condition (3.25), the amplitude (5.12) becomes

$$\mathcal{A}_{C_\theta \overline{C}_\theta U^j} = \frac{1}{\pi i} \frac{\overline{D}^j}{U^j - \overline{U}^j} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \mathcal{E} (z_1 - z_2)^{-2} (\overline{z}_3 - z_3)^{-2} (X_1 - 2 t X_2), \tag{5.14}$$

¹⁶ In this section we are in the type *IIA* picture of intersecting $D6$ -branes, where we however omit the prime on the moduli fields U^j .

with:

$$\begin{aligned}
X_1 &= -\frac{(\bar{z}_3 - z_1)^{-\theta^j} (z_3 - z_2)^{-\theta^j}}{(\bar{z}_3 - z_2)^{1-\theta^j} (z_3 - z_1)^{1-\theta^j}} \left[(z_3 - z_2)(\bar{z}_3 - z_1) + (1 - \theta^j)(z_3 - \bar{z}_3)(z_1 - z_2) \right] , \\
X_2 &= \left[\frac{(\bar{z}_3 - z_1)(z_3 - z_2)}{(\bar{z}_3 - z_2)(z_3 - z_1)} \right]^{1-\theta^j} .
\end{aligned} \tag{5.15}$$

As in the case with the untwisted matter field correlator, we fix three vertex positions according to (3.40) and include the c -ghost correlator (3.41). To this end, the string S -matrix $\mathcal{A}_{C_\theta \bar{C}_\theta U^j}$ may be expressed in terms of the integrals (3.43):

$$\begin{aligned}
\mathcal{A}_{C_\theta \bar{C}_\theta U^j} &= -\frac{i\bar{D}^j}{2\pi U_2^j} 2^{-4t-3} \left[(1+2t)I(-1-2t, 2\theta^j-2) - 4i(1-\theta^j)I(-2t, 2\theta^j-1) \right] \\
&= -\frac{i\bar{D}^j}{2U_2^j} e^{-i\pi\theta^j} \frac{\Gamma(-2t) \left[\Gamma(-t+\theta^j) + (1-\theta^j) \Gamma(-1-t+\theta^j) \right]}{\Gamma(-1-t+\theta^j) \Gamma(-t+\theta^j) \Gamma(1-t-\theta^j)} .
\end{aligned} \tag{5.16}$$

The above expression has the following momentum expansion:

$$\mathcal{A}_{C_\theta \bar{C}_\theta U^j} = -\frac{i\bar{D}^j}{4\pi U_2^j} e^{-i\pi\theta^j} \sin(\pi\theta^j) \left[1 + t \rho^j + \mathcal{O}(t^2) \right] , \tag{5.17}$$

where $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$ and

$$\rho^j = 2\gamma_E + \psi(\theta^j) + \psi(1-\theta^j) .$$

Using $D = e^{-2i\pi\theta^j}$, the symmetrized amplitude is finally

$$\mathcal{A}_{C_\theta \bar{C}_\theta U_2^j} = \frac{1}{2\pi U_2^j} \cos(\pi\theta^j) \sin(\pi\theta^j) \left[1 + t \rho^j + \mathcal{O}(t^2) \right] . \tag{5.18}$$

The linear term of the t -expansion (5.18) describes the derivative of the matter fields metric $G_{C_\theta \bar{C}_\theta}$ w.r.t. U_2^j . After redefining the vertex operators $V_{C_\theta} \rightarrow G_{C_\theta \bar{C}_\theta}^{1/2} V_{C_\theta}$, which converts the metric in the canonical normalized string basis into the field-theory basis, we obtain the following differential equation

$$\frac{\partial}{\partial U_2^j} G_{C_\theta \bar{C}_\theta} = -G_{C_\theta \bar{C}_\theta} \frac{1}{2\pi U_2^j} \cos(\pi\theta^j) \sin(\pi\theta^j) \rho^j . \tag{5.19}$$

Obviously, for all three complex planes $j = 1, 2, 3$, the scattering amplitude (5.12) yields the same type of equation. Through (3.22) the angle θ^j depends on the moduli U^i . From (3.20) we obtain

$$\frac{\partial \theta^j}{\partial U_2^j} = -\frac{1}{\pi U_2^j} \cos(\pi\theta^j) \sin(\pi\theta^j) . \tag{5.20}$$

With these relations, we may convert the equations (5.19) into differential equations w.r.t. θ^j :

$$\frac{\partial}{\partial \theta^j} \ln G_{C_\theta \overline{C}_\theta} = \frac{\rho^j}{2}. \quad (5.21)$$

These equations integrate to:

$$G_{C_\theta \overline{C}_\theta} \sim e^{\gamma_E \sum_{j=1}^3 \theta^j} \prod_{j=1}^3 \sqrt{\frac{\Gamma(\theta^j)}{\Gamma(1-\theta^j)}}. \quad (5.22)$$

Note, that this expression depends on the complex structure moduli U^j through the relation (3.22).

The dependence of the metric $G_{C_\theta \overline{C}_\theta}$ on the moduli t^i can be computed from the four-point disk diagram in the twisted sector, similarly as in the untwisted case. The relevant amplitude $\mathcal{A} = \langle V_{C_\theta} V_{t^m} V_{\overline{t}^m} V_{\overline{C}_\theta} \rangle$ is:

$$\mathcal{A} \sim \prod \int dx_i \mathcal{J}(x_i) \langle V_{C_\theta}^{-1}(k_1, x_1) V_{t^m}^0(k_2, x_2; k_3, x_3) V_{\overline{t}^m}^0(k_4, x_4; k_5, x_5) V_{\overline{C}_\theta}^{-1}(k_6, x_6) \rangle. \quad (5.23)$$

We proceed exactly as in the untwisted case and find

$$\begin{aligned} \mathcal{A} &\sim \int \frac{dw}{w} \int d^2 y |1-y|^{u-4} |y|^{t-2\theta'} \left\{ F(1, y) F(1, \overline{y}) - \frac{u}{4} [F(1, y) + F(1, \overline{y})] \right\} \\ &\sim \frac{ts}{u} + (1-\theta') s + \mathcal{O}(k^4), \end{aligned} \quad (5.24)$$

where $F(a, b) = (1-\theta') + \theta' \frac{b}{a}$ and $\theta' = 1 - \theta^m$. The relative coefficients of the two terms in the last line of the above equation determine the moduli dependence of the metric:

$$G_{C_\theta \overline{C}_\theta} = \tilde{G} \prod_m (t^m + \overline{t}^m)^{-\theta^m}. \quad (5.25)$$

Here \tilde{G} an arbitrary function of the other fields.

6. Scattering of four matter fields and normalization of Yukawa couplings

In the following we study 4-point functions of four matter fields

$$\mathcal{A}_{ij} = C_A \cdot \langle V_{C_i}(z_1) V_{\overline{C}_i}(z_2) V_{C_j}(z_3) V_{\overline{C}_j}(z_4) \rangle, \quad (6.1)$$

to obtain further information on the normalization of the physical Yukawa couplings and the holomorphic superpotential of the matter fields. Here C_i and C_j are again brane matter fields and C_A an yet undetermined normalization constant.

If all matter fields are charged under a common gauge group, the leading term in an momentum expansion of the factorization limit $z_1 \rightarrow z_2$ contains an s -pole $\sim \kappa = (t-u)/s$ from an exchange of a gauge boson. For appropriate normalization this exchange agrees with the same process in the effective field theory

$$\mathcal{A}_{ij}^\kappa = \lim_{z_2 \rightarrow z_1} \mathcal{A}_{ij}|_\kappa = g_A^2 G_{C_i \overline{C}_i} G_{C_j \overline{C}_j}, \quad (6.2)$$

where we omitted identical group theory and kinematic factors on both sides. The subscript $|_\kappa$ denotes the coefficient of the kinematics of the gauge boson exchange. Moreover, g_A is the gauge coupling of the exchanged gauge boson on the brane $A(i)$ on which the matter field C_i lives. The above equation may be used to eliminate the constant C_A in favor of the field theory quantities.

If the string amplitude has been normalized as in (6.2), the leading term in the factorization limit $z_3 \rightarrow z_4$ reproduces the contact term from the Yukawa couplings in the effective field theory

$$\mathcal{A}_{ij}^Y = \lim_{z_2 \rightarrow z_4} \mathcal{A}_{ij} = \sum_k Y_{ijk} G^{C_k \overline{C}_k} \overline{Y}_{\overline{k}ji} = e^{\kappa_4^2 \widehat{K}} \sum_k W_{ijk} G^{C_k \overline{C}_k} \overline{W}_{\overline{k}ji}. \quad (6.3)$$

Here we used the relation between the physical Yukawa couplings and the holomorphic superpotential, at cubic order in the matter fields [22][2],

$$Y_{\alpha\beta\gamma} = e^{\frac{1}{2}\kappa_4^2 \widehat{K}} W_{\alpha\beta\gamma}. \quad (6.4)$$

If W_{ijk} is “invertible” in the sense that one can chose the indices such that the sum over k collapses, the two equations imply

$$|W_{ijk}|^2 = e^{-\kappa_4^2 \widehat{K}} g_A^2 G_{C_i \overline{C}_i} G_{C_j \overline{C}_j} G_{C_k \overline{C}_k} \left(\frac{A_{ij}^Y}{A_{ij}^\kappa} \right), \quad (6.5)$$

which is independent of the normalization constant. The above equation relates the holomorphic superpotential and the Yukawa couplings in the string frame determined by A_{ij}^Y .

Note that W is holomorphic by definition and on general grounds we expect that for the, say, type *IIA* theory with branes at angles, it depends only on the Kähler moduli t^i [38]. In this way the two factorization limits discussed above lead to a non-trivial constraint on the expressions appearing on the r.h.s. For the Kähler potential (3.14), the exponential factor becomes

$$e^{-\kappa_4^2 \widehat{K}} = e^{-4\phi_4} \prod_i U_2^{IIB,i} = e^{-4\phi_4} \prod_i T_2^{IIA,i},$$

and is in fact independent of the geometric moduli $T(U)$ in the type *IIB* (*IIA*) theory.

6.1. Untwisted matter fields C_i

In the following we shall determine the scattering of four untwisted matter fields:

$$\begin{aligned} \mathcal{A}_{\overline{C}_i C_i \overline{C}_j C_j}(k_1, k_2, k_3, k_4) &= C \int_{-\infty}^{\infty} \prod_{i=1}^4 dz_i V_{\text{CKG}}^{-1} \\ &\times \langle V_{\overline{C}_i}^{(0)}(z_1, k_1) V_{C_i}^{(-1)}(z_2, k_2) V_{\overline{C}_j}^{(0)}(z_3, k_3) V_{C_j}^{(-1)}(z_4, k_4) \rangle . \end{aligned} \quad (6.6)$$

Two matter fields C_i, \overline{C}_i refer to the i -th subplane, whereas the other two fields C_j, \overline{C}_j originate from the j -th subplane. The constant C is the normalization of the four-point function to be determined later. All matter field vertex operators are inserted at the boundary of the disk. In the zero ghost picture they have been given in (4.2), and in the -1 ghost picture they read:

$$V_{C_i}^{(-1)}(z, k) = \lambda e^{-\phi(z)} \Psi^i(z) e^{ik_\nu X^\nu(z)} . \quad (6.7)$$

Due to internal charge conservation, a non-vanishing contribution arises in (6.6) only if all internal complex fermions are contracted. Therefore, we obtain:

$$\begin{aligned} \mathcal{A}_{\overline{C}_i C_i \overline{C}_j C_j}(k_i) &= -t C \int \prod_{i=1}^4 dz_i V_{\text{CKG}}^{-1} \mathcal{E} \\ &\times \frac{1}{z_2 - z_4} \frac{1}{z_1 - z_3} \left[\frac{1}{(z_1 - z_2)(z_3 - z_4)} + \frac{\delta^{ij}}{(z_2 - z_3)(z_1 - z_4)} \right] . \end{aligned} \quad (6.8)$$

The correlator \mathcal{E} for the exponentials is given in (3.38) and is subject to the momentum constraint (2.8). We may use $PSL(2, \mathbf{R})$ -invariance on the disk to fix three vertex positions

$$z_1 = 0 \quad , \quad z_2 = x \quad , \quad z_3 = 1 \quad , \quad z_4 := z_\infty = \infty , \quad (6.9)$$

and introduce the factor z_∞^2 to account for the c -ghosts. In that case the expression (6.8) reduces to

$$\begin{aligned} \mathcal{A}_{\overline{C}_i C_i \overline{C}_j C_j}(k_i) &= -t C \text{Tr}(\lambda_i^\dagger \lambda_i \lambda_j^\dagger \lambda_j) \int_0^1 dx x^s (1-x)^u \left(\frac{1}{x} + \delta^{ij} \frac{1}{1-x} \right) \\ &- t C \text{Tr}(\lambda_i^\dagger \lambda_j^\dagger \lambda_i \lambda_j) \int_0^1 dx x^{t-1} (1-x)^u \left(1 + \delta^{ij} \frac{1}{x-1} \right) \\ &- t C \text{Tr}(\lambda_i^\dagger \lambda_j^\dagger \lambda_j \lambda_i) \int_0^1 dx x^{t-1} (1-x)^s \left(\frac{1}{x-1} + \delta^{ij} \right) \end{aligned} \quad (6.10)$$

after taking into account the ordering of the Chan–Paton factors w.r.t. the vertex operator positions. With the Veneziano integral

$$V(s, u) := \int_0^1 dx x^s (1-x)^u = \frac{\Gamma(s+1) \Gamma(u+1)}{\Gamma(2+s+u)}, \quad (6.11)$$

we obtain:

$$\begin{aligned} \mathcal{A}_{\overline{C}_i C_i \overline{C}_j C_j}(k_i) &= -t C \operatorname{Tr}(\lambda_i^\dagger \lambda_i \lambda_j^\dagger \lambda_j) [V(s-1, u) + \delta^{ij} V(s, u-1)] \\ &\quad - t C \operatorname{Tr}(\lambda_i^\dagger \lambda_j^\dagger \lambda_i \lambda_j) [V(t-1, u) - \delta^{ij} V(t-1, u-1)] \\ &\quad - t C \operatorname{Tr}(\lambda_i^\dagger \lambda_j^\dagger \lambda_j \lambda_i) [-V(t-1, s-1) + \delta^{ij} V(t-1, s)] . \end{aligned} \quad (6.12)$$

Adding the group factors from the flipped diagrams and expanding up to fourth order in the momenta we finally arrive at

$$\begin{aligned} \mathcal{A}_{\overline{C}_i C_i \overline{C}_j C_j}(k_i) &= C \left[\operatorname{Tr}(\lambda_i^\dagger \lambda_i \lambda_j^\dagger \lambda_j) + \operatorname{Tr}(\lambda_i^\dagger \lambda_j \lambda_j^\dagger \lambda_i) \right] \left[\left(-\frac{t}{s} + \frac{\pi^2}{6} tu \right) + \delta^{ij} \left(-\frac{t}{u} + \frac{\pi^2}{6} st \right) \right] \\ &\quad + C \left[\operatorname{Tr}(\lambda_i^\dagger \lambda_j^\dagger \lambda_i \lambda_j) + \operatorname{Tr}(\lambda_i^\dagger \lambda_j \lambda_i \lambda_j^\dagger) \right] \left[\left(-1 + \frac{\pi^2}{6} tu \right) + \delta^{ij} \left(-\frac{s}{u} + \frac{\pi^2}{6} st \right) \right] \\ &\quad + C \left[\operatorname{Tr}(\lambda_i^\dagger \lambda_j^\dagger \lambda_j \lambda_i) + \operatorname{Tr}(\lambda_i^\dagger \lambda_i \lambda_j \lambda_j^\dagger) \right] \left[\left(-\frac{u}{s} + \frac{\pi^2}{6} tu \right) + \delta^{ij} \left(-1 + \frac{\pi^2}{6} st \right) \right] . \end{aligned} \quad (6.13)$$

The two factorization limits discussed above can now be read off from the general result (6.13) with $i \neq j$. More precisely, the above result implies that the ratio A_{ij}^Y/A_{ij}^κ is a constant, whereas it does not determine the index structure of the cubic couplings W_{ijk} . The latter can be inferred from the charge selection rules of the internal CFT, with the result that $W \neq 0$ only if $i \neq j \neq k$ [51]. Using the result (4.22) for the matter metric, and (3.17) for the gauge coupling, one finds that the r.h.s. of (6.5) is a constant, leading to a moduli independent superpotential

$$W = \text{const. } C_1^9 C_2^9 C_3^9.$$

A similar analysis can be performed for the other terms in the superpotential of [51], containing various couplings for 5- and 9-brane matter fields, and again one finds that the cubic terms in the superpotential are moduli independent. This is in contrast to formulae in the literature, where the cubic terms in W sometimes appear with extra factors of the brane gauge couplings.

6.2. Twisted matter fields C_θ

Finally we consider the scattering of four twisted matter fields:

$$\begin{aligned} \mathcal{A}_{C_\theta \bar{C}_\theta C_\nu \bar{C}_\nu}(k_i) &= C \int \prod_{i=1}^4 dz_i V_{\text{CKG}}^{-1} \\ &\times \langle V_{C_\theta}^{(0)}(z_1, k_1) V_{\bar{C}_\theta}^{(-1)}(z_2, k_2) V_{C_\nu}^{(0)}(z_3, k_3) V_{\bar{C}_\nu}^{(-1)}(z_4, k_4) \rangle, \end{aligned} \quad (6.14)$$

All matter field vertex operators are inserted at the boundary of the disk. The above choice of matter field vertices (5.3) and (5.8) in the -1 and 0 ghost picture, respectively has the advantage, that no contribution of excited twist or spin fields contribute due to internal $U(1)$ charge conservation. Hence from both vertex operators in the zero ghost picture (5.8) only the second term contributes. We immediately obtain the factor $t(z_1 - z_3)^{-1}$ from contracting the two space-time fermions $\psi(z_1)$ and $\psi(z_3)$. The contribution \mathcal{E} of the exponentials is given in (3.38), the ghosts give a factor $(z_2 - z_4)^{-1}$, and the four fermion correlator is

$$\langle s_{\theta^j}(z_1) s_{-\theta^j}(z_2) s_{\nu^j}(z_3) s_{-\nu^j}(z_4) \rangle = \frac{(z_1 - z_2)^{-(1-\theta^j)^2}}{(z_3 - z_4)^{(1-\nu^j)^2}} \left(\frac{z_{13}z_{24}}{z_{14}z_{23}} \right)^{(1-\theta^j)(1-\nu^j)}. \quad (6.15)$$

The bosonic twist correlators $Z_b = \langle \sigma_{-\theta^j}(z_1) \sigma_{\theta^j}(z_2) \sigma_{-\nu^j}(z_3) \sigma_{\nu^j}(z_4) \rangle$ are slightly more involved. The four twist correlation function can be computed by adapting the arguments of [43,42] to the open string case. In particular the result for generic fluxes/angles, but $\nu = \theta$, has been obtained in [48]. Since all twist fields are inserted at the boundary of the disk, the correlator for $\nu \neq \theta$ can be read off from the closed string correlator [44,45] by “taking the square root”:¹⁷

$$Z_b = \mathcal{C}_j (-1)^{2h_{\sigma_{\theta^j}} + 2h_{\sigma_{\nu^j}}} z_{12}^{-2h_{\sigma_{\theta^j}}} z_{34}^{-2h_{\sigma_{\nu^j}}} I_j(x)^{-1/2} Z_j^{cl} \left(\frac{z_{13}z_{24}}{z_{14}z_{23}} \right)^{\frac{1}{2}\theta^j + \frac{1}{2}\nu^j - \theta^j\nu^j}, \quad (6.16)$$

where $\mathcal{C}_j = \sqrt{\sin(\pi\theta)}$, $x = z_{12}z_{34}/z_{13}z_{24}$ and Z_j^{cl} denotes the instanton sum [48]. Moreover

$$I_j(x) = \frac{1}{\pi} \sin(\pi\theta^j) [B_{1;j} G_{2;j}(x) H_{1;j}(1-x) + B_{2;j} G_{1;j}(x) H_{2;j}(1-x)], \quad (6.17)$$

where $B_{1;j} = \frac{\Gamma(\theta^j)\Gamma(1-\nu^j)}{\Gamma(1+\theta^j-\nu^j)}$, $B_{2;j} = \frac{\Gamma(\nu^j)\Gamma(1-\theta^j)}{\Gamma(1+\nu^j-\theta^j)}$, $G_{1;j}(x) = {}_2F_1[\theta^j, 1-\nu^j, 1; x]$, $G_{2;j}(x) = {}_2F_1[1-\theta^j, \nu^j, 1; x]$, $H_{1;j}(x) = {}_2F_1[\theta^j, 1-\nu^j, 1+\theta^j-\nu^j; x]$, and $H_{2;j}(x) = {}_2F_1[1-\theta^j, \nu^j, 1-\theta^j+\nu^j; x]$. In total the contribution of the internal twist fields amounts to

$$(-1)^{-\sum_{j=1}^3 (\theta^j)^2 + (\nu^j)^2} \frac{z_{13}z_{24}}{z_{12}z_{14}z_{23}z_{34}} \prod_{j=1}^3 \mathcal{C}_j I_j(x)^{-1/2} Z_j^{cl} \quad (6.18)$$

¹⁷ See also [50,18,20] for a further discussion of the open string case.

due to (3.25). Putting everything together, the amplitude (6.14) becomes

$$\begin{aligned} \mathcal{A}_{C_\theta \bar{C}_\theta C_\nu \bar{C}_\nu}(k_i) &= \frac{t}{2} \int_{-\infty}^{\infty} \prod_{i=1}^4 dz_i V_{\text{CKG}}^{-1} \prod_{j=1}^3 \mathcal{C}_j I_j(x)^{-\frac{1}{2}} Z_j^{cl} \\ &\times z_{12}^{-\alpha' s-1} z_{13}^{-\alpha' t} z_{14}^{-\alpha' u-1} z_{23}^{-\alpha' u-1} z_{24}^{-\alpha' t} z_{34}^{-\alpha' s-1} . \end{aligned} \quad (6.19)$$

We fix three vertex positions according to (6.9) and introduce the factor z_∞^2 to account for the c -ghost correlator. With that choice (6.19) reduces to:

$$\mathcal{A}_{C_\theta \bar{C}_\theta C_\nu \bar{C}_\nu}(k_i) = \frac{t}{2} \int_{-\infty}^{\infty} dz z^{-\alpha' s-1} (z-1)^{-\alpha' u-1} \prod_{j=1}^3 \mathcal{C}_j I_j(z)^{-\frac{1}{2}} Z_j^{cl} . \quad (6.20)$$

In order to extract the Yukawa couplings, we investigate the limit $z \rightarrow \infty$ in the integrand of the four matter fields amplitude (6.20). After using the relation [44,45]

$$\lim_{z \rightarrow \infty} \pi^{-1} \sin(p\pi\theta^j) I_j(z)^{-1} \longrightarrow \begin{cases} (-1)^{k-l} z^{\theta^j + \nu^j} \Gamma_{\theta^j, \nu^j} & 0 < \theta^j + \nu^j < 1 , \\ -(-1)^{k-l} z^{2-\theta^j - \nu^j} \Gamma_{1-\theta^j, 1-\nu^j} & 1 < \theta^j + \nu^j < 2 , \end{cases} \quad (6.21)$$

with

$$\Gamma_{\theta^j, \nu^j} = \frac{\Gamma(1-\theta^j) \Gamma(1-\nu^j) \Gamma(\theta^j + \nu^j)}{\Gamma(\theta^j) \Gamma(\nu^j) \Gamma(1-\theta^j - \nu^j)} , \quad (6.22)$$

the integrand of (6.20) becomes in the limit $z \rightarrow \infty$:

$$\frac{t}{2} \prod_{j=1}^3 \Gamma_{\theta^j, \nu^j}^{1/2} |W_j|^2 \int_{-\infty}^{\infty} dz z^{-\alpha' t-1} \longrightarrow \prod_{j=1}^3 \Gamma_{\theta^j, \nu^j}^{1/2} |W_j|^2 . \quad (6.23)$$

Here the W_j is the superpotential describing the (classical) world-sheet disk instanton contributions elaborated in [17,18]. From the above one finds the relative factor

$$\prod_{j=1}^3 \left[\frac{\Gamma(1-\theta^j) \Gamma(1-\nu^j) \Gamma(\theta^j + \nu^j)}{\Gamma(\theta^j) \Gamma(\nu^j) \Gamma(1-\theta^j - \nu^j)} \right]^{1/4} . \quad (6.24)$$

between the Yukawa couplings in the string and the field theory basis, in agreement with the results of section 5. Note that the correct power 1/4 of the above factor is in contrast to a power 1/2 claimed in Refs. [18,19].¹⁸

¹⁸ As mentioned already, the quantum part encoded in Γ_{θ^j, ν^j} is at any rate just the square root of the closed string calculation, and can therefore be read off from *Eq.* (4.16) of [45].

7. Conclusions

For type *IIB* orbifold/orientifold compactifications with *D9*, *D7*, *D5* and *D3*-branes with internal background fluxes we have calculated the complete (bosonic) tree-level action up to second order in the space-time momenta and scalar matter fields. Our findings directly apply to type *I* orbifold/orientifold compactifications with *D9* and *D5*-branes. Moreover, after performing *T*-duality (3.18), our deliverables take over to type *IIA* orbifold/orientifold compactifications with intersecting *D6*-branes. Although our outcomes hold also for the non-supersymmetric case, here let us review those for the $N=1$ supersymmetric case in $D = 4$. We collect the main formulae for type *IIB* orbifold/orientifold compactifications with *D9*-branes with non-trivial background fluxes.

In section 3 the holomorphic gauge kinetic function f has been determined via disk scattering of two gauge fields and one modulus¹⁹ with the result (3.17):

$$f(s, t^j) = |n^1 n^2 n^3| \left(s - \alpha'^{-2} f^1 f^2 t^3 - \alpha'^{-2} f^1 f^3 t^2 - \alpha'^{-2} f^2 f^3 t^1 \right). \quad (7.1)$$

Here, f^j are three background fluxes on the *D9*-brane, referring to the three tori $T^{2,j}$ the *D9*-brane is wrapped on with the wrapping numbers n^j .

The Yukawa couplings and the superpotential were derived by computing open string four point amplitudes on the disk. Further details may be looked up in section 6.

The Kähler potential for the untwisted fields has been derived in section 4 (see also the appendix) and is summarized in (4.22), while the metric for the general case has been determined in section 5. For completeness we discuss here also the special case of matter fields with one angle equal to zero, which appear in the context the *T*-dual of pure $2p$ -branes. Although the metric for these cases does not, in general, follow from the generic formula due to the appearance of extra massless states, the computation can be straightforwardly adapted. For pure 5 and 9 branes one finds

$$G_{C^{95_i} \overline{C}^{95_i}} = \frac{1}{(t_1^j t_1^k u_1^j u_1^k)^{\frac{1}{2}}}, \quad G_{C^{5_i 5_j} \overline{C}^{5_i 5_j}} = \frac{1}{(s_1 t_1^k u_1^i u_1^j)^{\frac{1}{2}}}, \quad (7.2)$$

which again agrees with the expressions of [34] in the case of fixed complex structure.

We have determined the Kähler potential for the metric moduli T^j, U^j and matter field moduli C_i, C_θ in the absence of Wilson line moduli. It would be very interesting to extend our calculation to the case, when Wilson lines are turned on. The moduli space of metric and matter fields of $D = 4$ Calabi-Yau compactifications without branes shows some restricted structure, known as special geometry [2]. This is the case for heterotic

¹⁹ One should mention at this point that the string computation of [48] already implicitly contained this result.

N=1 Calabi–Yau compactifications and follows from the underlying $(2, 2)$ superconformal world-sheet symmetry, which governs the closed string vertex operators. A natural question is, whether the geometry of the moduli space of N=1 type *I* or type *II* compactifications involving both open and closed strings (and *D*–branes) shows a similar restricted structure. To address this questions, one has to calculate four point–disk scattering amplitudes involving both open and closed strings to obtain conditions on the Riemann tensor of the moduli space.

Finally, the discrete symmetries acting on the moduli fields imply non–trivial transformations on the various vertex operators for the metric and matter field moduli, quite similar to the heterotic case [52,53,45]. It would be interesting to study those transformation properties within these type *I* and type *II* vacua, used in this paper.

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Appendix A. Scattering of two closed string moduli off the brane

Finally, in this section we shall determine the function $\hat{K}(M, \overline{M})$ in Eq. (1.1) through string amplitudes. This term describes the metric of the Kähler and complex structure moduli T^j, U^j , respectively. We consider the scattering of two closed string moduli fields T^j, U^j on the disk. We perform our calculation on the type *IIB* side with general fluxes \mathcal{F}^j on the *D9*–brane turned on.

A.1. Disk amplitudes with two closed strings

Following the notation introduced in section 2 we consider the case $N_o = 0$, $N_c = 2$. As we have already reviewed there, a closed string vertex operator inserted on the disk \mathcal{H}_+ is split into two open string vertex operators depending respectively holomorphically or anti–holomorphically on the complex sphere coordinate z . Their interactions are described by the correlator (2.2) with the matrix D^{ij} . Again the closed string momentum q is distributed to the two open strings according to (2.5). Hence, two closed string vertex operators with momenta q_1, q_2 on the disk are treated as four open strings of momenta $\frac{1}{2}q_1, \frac{1}{2}Dq_1, \frac{1}{2}q_2$ and $\frac{1}{2}Dq_2$, respectively. Only the momentum parallel to the *D*–brane is conserved:

$$q_1^{\parallel} + q_2^{\parallel} = 0 . \tag{A.1}$$

After introducing the four momenta k_i entering in the four open string vertex operators

$$k_1 = q_1 \quad , \quad k_2 = Dq_1 \quad , \quad k_3 = q_2 \quad , \quad k_4 = Dq_2 \quad (\text{A.2})$$

Eq. (A.1) translates into (2.8), with $k_i^2 = 0$. Again, we may introduce the Mandelstam variables (2.9), with $s + t + u = 0$. However, now *Eq.* (2.9) implies the constraints [26]

$$s = 2 (q_1^\parallel)^2 = 2 (q_2^\parallel)^2 = -2 q_1^\parallel q_2^\parallel \quad , \quad t = q_1 q_2 \quad , \quad (\text{A.3})$$

in contrast to (2.10) for the case $N_o = 2$, $N_c = 1$.

A.2. Disk scattering of two metric moduli fields

We shall first consider the scattering of two Kähler moduli T^i and \bar{T}^j on the disk:

$$\mathcal{A}_{T^i \bar{T}^j}(q_1, q_2) = C \int d^2 z_1 d^2 z_2 V_{\text{CKG}}^{-1} \langle V_{T^i}^{(0,0)}(\bar{z}_1, z_1, q_1) V_{\bar{T}^j}^{(-1,-1)}(\bar{z}_2, z_2, q_2) \rangle . \quad (\text{A.4})$$

There are two contributions, one X_1 coming from the contraction of only two internal fermions and a second X_2 from contracting all four internal fermions:

$$\begin{aligned} \mathcal{A}_{T^i \bar{T}^j}(q_1, q_2) = C & \frac{1}{(T^i - \bar{T}^i)(T^j - \bar{T}^j)} \int d^2 z_1 d^2 z_2 V_{\text{CKG}}^{-1} \frac{\mathcal{E}}{\bar{z}_2 - z_2} \\ & \times \left\{ \frac{D^j \bar{D}^i}{(\bar{z}_1 - z_1)^2 (\bar{z}_2 - z_2)} - \frac{s}{\bar{z}_1 - z_1} \left[\frac{D^j \bar{D}^i}{(\bar{z}_1 - z_1)(\bar{z}_2 - z_2)} - \frac{\delta^{ij}}{(\bar{z}_1 - \bar{z}_2)(z_1 - z_2)} \right] \right\} . \end{aligned} \quad (\text{A.5})$$

On the disk the conformal Killing volume V_{CKG}^{-1} is canceled by fixing three positions and introducing the respective ghost correlator. A convenient choice of vertex operator positions is [27]

$$z_1 = iy \quad , \quad \bar{z}_1 = -iy \quad , \quad z_2 = i \quad , \quad \bar{z}_2 = -i \quad , \quad (\text{A.6})$$

which implies the ghost factor $1 - y^2$. With this choice the amplitude $\mathcal{A}_{T^i \bar{T}^j}$ becomes²⁰:

$$\begin{aligned} \mathcal{A}_{T^i \bar{T}^j}(q_1, q_2) = \frac{C}{4} & \frac{1}{(T^i - \bar{T}^i)(T^j - \bar{T}^j)} \\ & \times \int_0^1 dy \left[\frac{4y}{(1+y)^2} \right]^s \left(\frac{1-y}{1+y} \right)^{2t} \left[D^j \bar{D}^i \frac{1-s}{4} \frac{1-y^2}{y^2} - \delta^{ij} s \frac{1+y}{y(1-y)} \right] . \end{aligned} \quad (\text{A.7})$$

²⁰ With this choice (A.6), the exponential \mathcal{E} in (3.38) becomes $\mathcal{E} = \left| \frac{4y}{(1+y)^2} \right|^s \left| \frac{1-y}{1+y} \right|^{2t}$.

Now we perform the same steps as described in [27] to bring the above integral into the standard form (6.11):

$$\begin{aligned}\mathcal{A}_{T^i \bar{T}^j}(q_1, q_2) &= \frac{1}{(T^i - \bar{T}^i)(T^j - \bar{T}^j)} \left[D^j \bar{D}^i (s-1) V(t, s-2) + \delta^{ij} s V(t-1, s-1) \right] \\ &= \frac{1}{(T^i - \bar{T}^i)(T^j - \bar{T}^j)} \left(D^j \bar{D}^i t + \delta^{ij} s \right) (1-s) \frac{\Gamma(t)\Gamma(s-1)}{\Gamma(s+t)} .\end{aligned}\tag{A.8}$$

With the identity $D^i \bar{D}^i = 1$ we obtain for the case $i = j$

$$\begin{aligned}\mathcal{A}_{T^i \bar{T}^i}(q_1, q_2) &= -\frac{1}{(T^i - \bar{T}^i)^2} (s+t)^2 \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} \\ &= -\frac{1}{(T^i - \bar{T}^i)^2} \left(1 - \frac{\pi^2}{6} st \right) \frac{(s+t)^2}{st} + \mathcal{O}(k^6) ,\end{aligned}\tag{A.9}$$

whereas the case $i \neq j$ gives:

$$\begin{aligned}\mathcal{A}_{T^i \bar{T}^j}(q_1, q_2) &= -\frac{D^j \bar{D}^i}{(T^i - \bar{T}^i)(T^j - \bar{T}^j)} \frac{\Gamma(s)\Gamma(t+1)}{\Gamma(s+t)} \\ &= -\frac{D^j \bar{D}^i}{(T^i - \bar{T}^i)(T^j - \bar{T}^j)} \left(1 - \frac{\pi^2}{6} st \right) \frac{s+t}{s} + \mathcal{O}(k^6) .\end{aligned}\tag{A.10}$$

Note, that the first result (A.9) is completely independent on the value of the fluxes f^j , since the parameter D^i has dropped out. Only in that case we find a term proportional to s/t to conclude

$$G_{T^i \bar{T}^j} = -\frac{\delta^{ij}}{(T^i - \bar{T}^i)^2} .\tag{A.11}$$

That pole-term describes a pole on the sphere with the closed string vertices coming close together. Since the parameters D_i, \bar{D}_i , which determine the mixing of the open string boundary conditions on the $D9$ -brane, have completely dropped out for $\mathcal{A}_{T^i \bar{T}^i}$, the result is independent, whether the internal open string fields of the vertex operators V_{T^i} and $V_{\bar{T}^i}$ obey Dirichlet, Neumann or mixed boundary conditions. This means, that we would have anticipated the same result for $\mathcal{A}_{T^i \bar{T}^i}$ from scattering T^i and \bar{T}^i in the presence of a $D5$ -brane wrapped around a torus $T^{2,k}$, with $k \neq i$. In that case the torus $T^{2,i}$ would be transversal to the $D5$ -brane with the open string fields obeying Dirichlet boundary conditions.

Let us now repeat our previous calculation for the disk scattering of two complex structure moduli U^i and \bar{U}^j . The details are similar to the above derivation. Again, one has to use the identity $\bar{D}^i D^i = 1$ to arrive at:

$$\mathcal{A}_{U^i \bar{U}^j}(q_1, q_2) = -\frac{\delta^{ij}}{(U^i - \bar{U}^i)^2} s \frac{\Gamma(t)\Gamma(s+1)}{\Gamma(1+s+t)} = \frac{\delta^{ij}}{(U^i - \bar{U}^i)^2} \frac{s}{t} \left(1 - \frac{\pi^2}{6} st \right) + \mathcal{O}(k^6) .\tag{A.12}$$

From this amplitude one concludes, that there is no coupling between complex structure moduli from different planes. Furthermore the Kähler metric $G_{U^i \bar{U}^i}$ for the complex structure moduli is encoded in the pole:

$$G_{U^i \bar{U}^j} = -\frac{\delta^{ij}}{(U^i - \bar{U}^i)^2} . \quad (\text{A.13})$$

Finally, the disk scattering of one Kähler modulus T^i and one complex structure modulus \bar{U}^j gives a vanishing result due to internal charge conservation:

$$\mathcal{A}_{T^i \bar{U}^j}(q_1, q_2) = 0 . \quad (\text{A.14})$$

The above scattering results, *i.e.* the metrics (A.11), (A.13) and (A.14), can be derived formally²¹ from Kähler potential for the metric moduli:

$$-\ln(S - \bar{S}) - \sum_{j=1}^3 \ln(T^j - \bar{T}^j) - \sum_{j=1}^3 \ln(U^j - \bar{U}^j) . \quad (\text{A.15})$$

The first term describes the kinetic energy term for the dilaton field S , with $\text{Im}(S) = e^{-2\phi_4}$ (*cf.* Appendix B). Alternatively, in terms of the *physical* moduli fields, introduced in Eqs. (3.12), (3.13) and (3.27), the Kähler potential \hat{K} without the matter field metrics may be also written (*cf.* (3.14)):

$$\hat{K}(M, \bar{M}) = -\kappa_4^{-2} \ln(s + \bar{s}) - \kappa_4^{-2} \sum_{j=1}^3 \ln(t^j + \bar{t}^j) - \kappa_4^{-2} \sum_{j=1}^3 \ln(u^j + \bar{u}^j) . \quad (\text{A.16})$$

To conclude, the Kähler potential (A.16) takes the same form as in ordinary type *I* or type *IIB* orientifold/orbifold compactifications without fluxes. For those models without fluxes Eq. (A.16) has been derived based on symmetry arguments in [34]. The fields entering in (A.16) stem from the untwisted sector, which explains its quite universal appearance even in the presence of fluxes. As we have seen in our calculations, it essentially makes no difference, whether we consider disk scattering of Kähler and complex structure moduli fields in the presence of a *D9*-brane or *D5*-brane. In other words, the result is independent, whether the torus, to which the moduli fields refer, is parallel or transverse to the *Dp*-brane. In fact, we could just stick to the bulk of the *Dp*-brane, where only the closed string sector of the underlying type *IIB* orbifold/orientifold lives. From scattering of four moduli fields on the sphere one ends up with the same result (A.16). It has been already pointed out in [2], that $D = 4$ heterotic $N=1$ or $N=2$ type *II* compactification on the same manifold have the same metric moduli spaces up to second order in the momenta. We have checked this for the kind of models we discuss in this article.

²¹ It should be kept in mind, that the scalars S, T^j and U^j do not represent scalars of $N=1$ chiral multiplets.

Appendix B. Matter and gauge fields coupling to the graviton and dilaton

In this section we shall be interested in the dilaton dependence of the gauge couplings and matter field metrics, which have been the subject of the previous sections. This discussion is necessary in order to obtain the dilaton dependence of the metrics. The vertex operator for the bosonic massless closed string modes describing a graviton, dilaton or anti-symmetric tensor in the $(-1, -1)$ ghost picture is given by:²²

$$V_G^{(-1, -1)}(\bar{z}, z, q) = \epsilon_{\mu\nu} e^{-\tilde{\phi}(\bar{z})} e^{-\phi(z)} \tilde{\psi}^\mu(\bar{z}) \psi^\nu(z) e^{iq_\nu X^\nu(\bar{z}, z)} . \quad (\text{B.1})$$

The polarization tensor $\epsilon_{\mu\nu}$ is subject to the on-shell conditions $\epsilon_{\mu\nu} q^\mu = 0 = \epsilon_{\mu\nu} q^\nu$ and $q^2 = 0$. Apart from these constraints we shall perform the calculation for arbitrary polarization $\epsilon_{\mu\nu}$, thus allowing to also extract the gauge and matter field couplings to the graviton and anti-symmetric tensor. The polarization $\epsilon_{\mu\nu}$ in (B.1) determines the relevant closed string state²³:

$$\begin{aligned} \epsilon_{\mu\nu} &= \epsilon_{\nu\mu} \quad , \quad \text{Graviton} , \\ \epsilon_{\mu\nu} &= -\epsilon_{\nu\mu} \quad , \quad \text{Kalb - Ramond} , \\ \epsilon_{\mu\nu} &= \frac{1}{\sqrt{2}} (\eta_{\mu\nu} - q_\mu \bar{q}_\nu - q_\nu \bar{q}_\mu) \quad , \quad \bar{q}^2 = 0 \quad , \quad q_\mu \bar{q}^\mu = 1 \quad , \quad \text{Dilaton} . \end{aligned} \quad (\text{B.2})$$

We consider disk amplitudes with one massless bosonic supergravity NS field inserted in the bulk and two open string states, inserted at the boundary of the disk. The open string states are respectively untwisted matter fields C_i , twisted matter fields C_θ , and gauge fields A_μ^a . First, we shall calculate the string S -matrix:

$$\mathcal{A}_{C_i \bar{C}_i G} = \frac{-i}{\pi} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \langle V_{C_i}^{(0)}(z_1, k_1) V_{\bar{C}_i}^{(0)}(z_2, k_2) V_G^{(-1, -1)}(\bar{z}_3, z_3, q) \rangle . \quad (\text{B.3})$$

The untwisted matter field vertex operators have been given in (4.2).

In (B.3) two contractions are possible: The first, denoted by X_1 , with the two internal bosons from the matter vertices contracted and a second X_2 with their two internal fermions contracted. All correlators are basic and may be looked up from section 2. To this end we find

$$\mathcal{A}_{C_i \bar{C}_i G} = \frac{-i}{\pi} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \mathcal{E} (X_1 + X_2) , \quad (\text{B.4})$$

²² For further details, see also [54].

²³ Note, that in type I or type IIB orientifolds the space-time anti-symmetric tensor $b_{\mu\nu}$ is generically projected out.

with

$$\begin{aligned}
X_1 &= -\text{Tr}(\epsilon) \frac{1}{(z_1 - z_2)^2} \frac{1}{(\bar{z}_3 - z_3)^2} , \\
X_2 &= \frac{1}{(z_1 - z_2)(\bar{z}_3 - z_3)} \left[\frac{s \text{Tr}(\epsilon)}{(z_1 - z_2)(\bar{z}_3 - z_3)} - \frac{k_1 \epsilon k_2}{(z_1 - \bar{z}_3)(z_2 - z_3)} + \frac{k_1 \epsilon^t k_2}{(z_1 - z_3)(z_2 - \bar{z}_3)} \right] ,
\end{aligned} \tag{B.5}$$

and the correlator (3.38). After fixing the vertex positions like (3.40) and introducing the ghost correlator (3.41), we obtain:

$$\begin{aligned}
\mathcal{A}_{C_i \bar{C}_i G} &= 2^{-4t-1} \pi^{-1} \left[\frac{1}{4} \text{Tr}(\epsilon) (1+2t) I(-1-2t, 0) + i k_1^t (\epsilon + \epsilon^t) k_2 I(-2t, 1) \right] \\
&= 2^{-2t-1} \pi^{-1/2} \left[\frac{1}{2} \text{Tr}(\epsilon) (1+2t) \frac{\Gamma(-\frac{1}{2}-t)}{\Gamma(-t)} - k_1^t (\epsilon + \epsilon^t) k_2 \frac{\Gamma(\frac{1}{2}-t)}{\Gamma(1-t)} \right] \\
&= \frac{1}{2} [t \text{Tr}(\epsilon) - t k_1^t (\epsilon + \epsilon^t) k_2 + \mathcal{O}(t^2)] .
\end{aligned} \tag{B.6}$$

The first term gives rise to the coupling of the two untwisted matter fields C_i, \bar{C}_i to the dilaton, while the second term describes their coupling to the graviton (at zero momentum). Obviously, there is no coupling of the anti-symmetric tensor $b_{\mu\nu}$ to the matter fields.

Next, we calculate the disk amplitude

$$\mathcal{A}_{C_\theta \bar{C}_\theta G} = \frac{-i}{\pi} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \langle V_{C_\theta}^{(0)}(z_1, k_1) V_{\bar{C}_\theta}^{(0)}(z_2, k_2) V_G^{(-1, -1)}(\bar{z}_3, z_3, q) \rangle . \tag{B.7}$$

involving two twisted matter fields C_θ, \bar{C}_θ inserted at the boundary of the disk and one closed string modulus from the bulk. The latter describes a massless bosonic member of the supergravity multiplet. It proves to be convenient to work with the matter vertices in the zero-ghost picture (5.8) and the closed string vertex (B.1). Since, the bosonic and fermionic twist fields in (B.7) completely decouple from the closed string vertex they correlators may be determined independently. Due to internal charge conservation they boil down to products of the basic Green's functions (5.5) and (5.11):

$$\begin{aligned}
&\langle [\sum_{l=1}^3 \tau_{-\theta^l} t_{\theta^l}(z_1) \prod_{\substack{j=1 \\ j \neq l}}^3 \sigma_{-\theta^j} s_{\theta^j}(z_1)] [\sum_{m=1}^3 \tau_{\theta^m} t_{-\theta^m}(z_2) \prod_{\substack{k=1 \\ k \neq m}}^3 \sigma_{\theta^k} s_{-\theta^k}(z_2)] \rangle = \frac{1}{(z_1 - z_2)^2} , \\
&\langle [\prod_{j=1}^3 \sigma_{-\theta^j} s_{\theta^j}(z_1)] [\prod_{k=1}^3 \sigma_{\theta^k} s_{-\theta^k}(z_2)] \rangle = \frac{1}{z_1 - z_2}
\end{aligned} \tag{B.8}$$

The contraction of the space-time fields in (B.7) are the same as in (B.3). In fact, it is not hard to see, that the amplitude $\mathcal{A}_{C_\theta \bar{C}_\theta G}$ takes the same form as (B.4) and (B.5). Hence we conclude:

$$\mathcal{A}_{C_\theta \bar{C}_\theta G} = \mathcal{A}_{C_i \bar{C}_i G} . \tag{B.9}$$

Finally, the coupling of the dilaton to two gauge fields at the boundary of the disk may be directly taken from [26]:

$$\begin{aligned}\mathcal{A}_{A^a A^a G} &= -2 \operatorname{Tr}(\epsilon) \left[(p_1 \xi_2)(p_2 \xi_1) - (p_1 p_2)(\xi_1 \xi_2) \right] t \frac{\Gamma(-2t)}{\Gamma(1-t)^2} \\ &= \operatorname{Tr}(\epsilon) \left[(p_1 \xi_2)(p_2 \xi_1) - (p_1 p_2)(\xi_1 \xi_2) \right] [1 + \mathcal{O}(t)] .\end{aligned}\quad (\text{B.10})$$

Hence, we conclude, that both the matter metrics and the gauge couplings have universal couplings to the dilaton at disk tree-level.

Appendix C. Disk scattering with excited twist fields

Let us now calculate the string S -matrix

$$\mathcal{A}_{C_\theta \bar{C}_\theta U^j} = \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \langle V_{C_\theta}^{(0)}(z_1, k_1) V_{\bar{C}_\theta}^{(0)}(z_2, k_2) V_{U^j}^{(-1, -1)}(\bar{z}_3, z_3, q) \rangle \quad (\text{C.1})$$

on the disk to extract information on the metric of two matter fields. Furthermore, the two matter field vertices are inserted at the boundary of the disk, *i.e.* $z_1 = \bar{z}_1, z_2 = \bar{z}_2$, while the closed string vertex operator for the U^j -modulus, given in (3.30), is inserted in the bulk. The latter is chosen in the $(-1, -1)$ ghost picture, while the matter vertices are taken in the 0 ghost picture (5.8) in order to guarantee a total ghost charge of -2 on the disk. Due to internal charge conservation, there are two non-vanishing contributions to the amplitude (5.12): One from the contraction of the internal fermions of the U^j -modulus with the both excited twist fields

They arise from the contraction of the U^j -vertex operator with either only the first terms of the matter vertices (5.8) or with only the second terms. We shall denote these two possibilities by X_1 and X_2 , respectively. The contraction of the exponentials is given in (3.38), *i.e.* by \mathcal{E} . The correlators (5.5), (5.5) are needed, together with the correlator:

$$\langle t_{\theta^j}(z_1) t_{-\theta^j}(z_2) \tilde{\Psi}^j(\bar{z}_3) \bar{\Psi}^j(z_3) \rangle = \frac{\bar{D}^j}{\bar{z}_3 - z_3} (z_1 - z_2)^{-\theta_j^2} \left[\frac{(z_1 - z_3)(z_2 - \bar{z}_3)}{(z_1 - \bar{z}_3)(z_2 - z_3)} \right]^{\theta_j} \quad (\text{C.2})$$

for the first contraction, while for the second contraction we use (5.10). We obtain

$$\mathcal{A}_{C_\theta \bar{C}_\theta U^j} = \frac{\bar{D}^j}{U^j - \bar{U}^j} \int \frac{dz_1 dz_2 d^2 z_3}{V_{\text{CKG}}} \mathcal{E} (z_1 - z_2)^{-2} (\bar{z}_3 - z_3)^{-2} (X_1 - 2t X_2) , \quad (\text{C.3})$$

with:

$$\begin{aligned}X_1 &= (1 - \theta^j) \left[\frac{(\bar{z}_3 - z_1)(z_3 - z_2)}{(z_3 - z_1)(\bar{z}_3 - z_2)} \right]^{-\theta^j} + \theta^j \left[\frac{(\bar{z}_3 - z_1)(z_3 - z_2)}{(z_3 - z_1)(\bar{z}_3 - z_2)} \right]^{1-\theta^j} , \\ X_2 &= - \left[\frac{(\bar{z}_3 - z_1)(z_3 - z_2)}{(z_3 - z_1)(\bar{z}_3 - z_2)} \right]^{1-\theta^j} .\end{aligned}\quad (\text{C.4})$$

The two terms X_1 and X_2 agree with the expressions, given in (5.15). Hence, the string S -matrix (C.1) gives the same result as the amplitude (5.12).

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